# Crosspolarization and Depolarization in Ellipsometry for Inhomogeneous Samples

- Introduction and Motivation
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  - **Spatial**
- Depolarization
  - Temporal
- Continuity Conditions at Inner Boundaries
- Crosspolarization
- **Depolarization**
  - **Spatial**

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Paul Dirac „*Quantum Mechanics*“: ~ 1935

***“each photon then interferes only with itself. Interference between different photons never occurs.”***

*Albert Einstein to his friend Michael Besso 1954:*

***“All these fifty years of conscious brooding have brought me no nearer to the answer to the question, 'What are light quanta?' Nowadays every Tom, Dick and Harry thinks he knows it, but he is mistaken.”***

R. Feynman QED- the strange theory matter and light, 1985

***When a photon comes down, it interacts with electrons throughout the glass, not just on the surface. The photon and electrons do some kind of dance, the net result of which is the same as if the photon hit only on the surface.***

W.E. Lamb 1995, Appl. Phys B

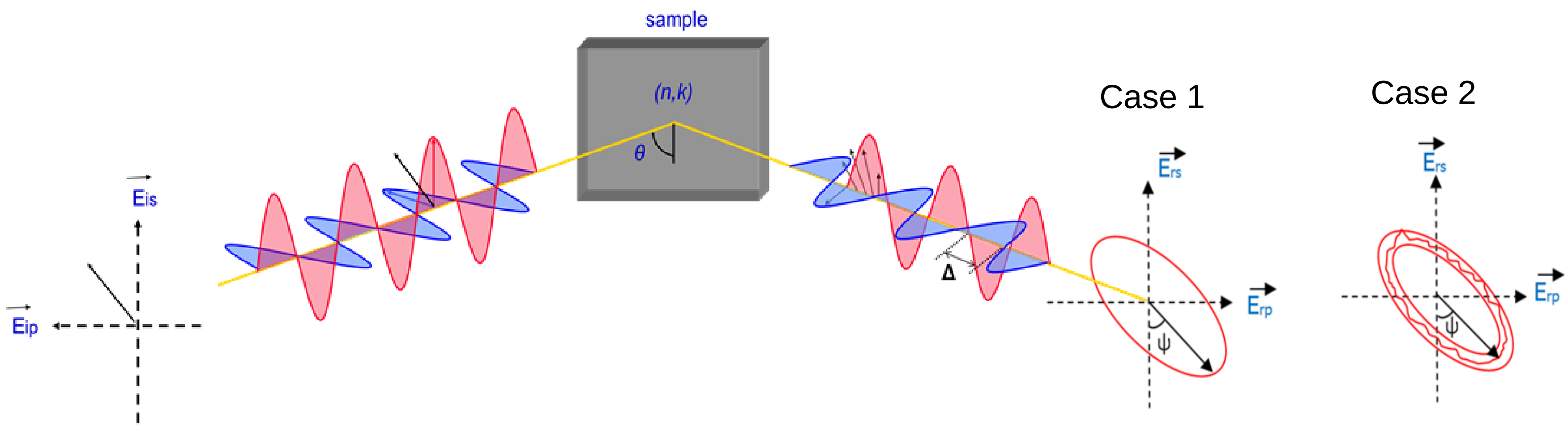
***“Photons cannot be localized in any meaningful manner, and they do not behave at all like particles, whether described by a wave function or not."***

Roy Glauber „Nobel lecture: *Quantum Mechanics*“: 2005

***“….if we get a click in a detector, we know that at that very moment the photon is just there….”***

# What does depolarization mean?

- Partial state of polarization produced by the interaction of polarized light and an optical element (depolarizer).

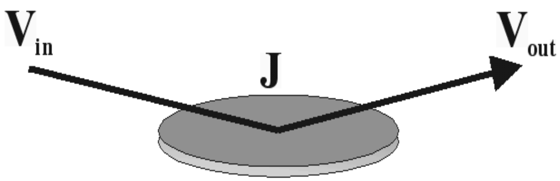


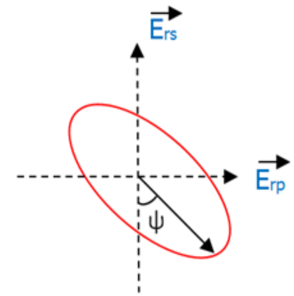

Two phase system: semi-infinite material & air

$$\rho = \frac{\left(\frac{\tilde{E}_r}{\tilde{E}_i}\right)_p}{\left(\frac{\tilde{E}_r}{\tilde{E}_i}\right)_s} = \frac{r_p}{r_s} = \frac{\sin^2(\theta) - \cos(\theta)\sqrt{\frac{\varepsilon_s}{\varepsilon_a} - \sin^2(\theta)}}{\sin^2(\theta) + \cos(\theta)\sqrt{\frac{\varepsilon_s}{\varepsilon_a} - \sin^2(\theta)}} =: \tan\psi\, e^{i\Delta}$$

# Case 1: For nondepolarising system fully equivalent

## Jones formalism



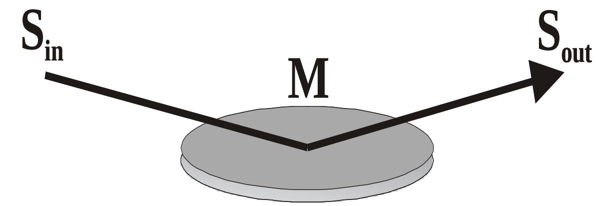

$$\mathbf{v}_{\mathbf{out}} = \mathbf{J}\,\mathbf{v}_{\mathbf{in}} \qquad \mathbf{v} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\begin{pmatrix} E_x^{out} \\ E_y^{out} \end{pmatrix} = \begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix} \cdot \begin{pmatrix} E_x^{in} \\ E_y^{in} \end{pmatrix}$$

## Common states of polarization

| H | V | 45° | -45° | CD | CG |
|---|---|---|---|---|---|
| $\begin{bmatrix}1\\0\end{bmatrix}$ | $\begin{bmatrix}0\\1\end{bmatrix}$ | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$ | $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$ |

## Mueller matrix formalism




$$\mathbf{S}_{\mathbf{out}} = \mathbf{M}\,\mathbf{S}_{\mathbf{in}}$$

$$\begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}_{\text{out}} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \cdot \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}_{\text{in}}$$

## Degree of polarization (DOP)

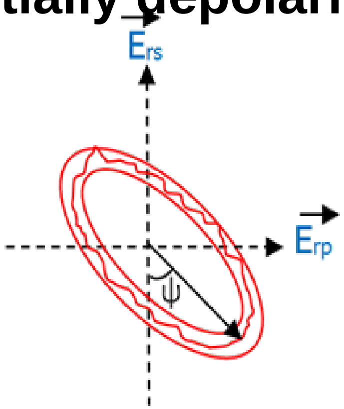
$$DOP = \frac{\sqrt{s_1^{\,2} + s_2^{\,2} + s_3^{\,2}}}{s_0} \qquad (0 \le DOP \le 1)$$

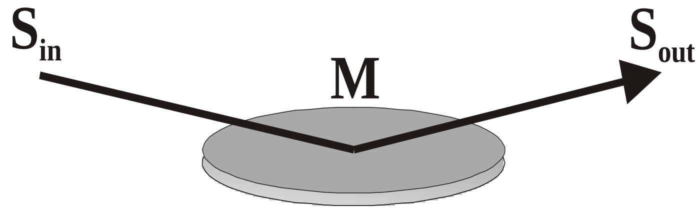
*Degree of Depolarization* (*approximately*)

$$\frac{s_0 - \sqrt{s_1^{\,2} + s_2^{\,2} + s_3^{\,2}}}{s_0}$$

**Totally polarized** $DOP = 1$

# Partially depolarizing case:




$$0 < DOP < 1$$

Partially polarized light

$$\mathbf{S}_{out} = \mathbf{M}\,\mathbf{S}_{in}$$

Stokes vector

$$\mathbf{S} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I_x + I_y \\ I_x - I_y \\ I_{45^\circ} - I_{-45^\circ} \\ I_G - I_D \end{pmatrix} = \begin{pmatrix} < E_x E_x^* + E_y E_y^* > \\ < E_x E_x^* - E_y E_y^* > \\ < E_x E_y^* + E_y E_x^* > \\ i < E_x E_y^* - E_y E_x^* > \end{pmatrix}$$

$$\mathbf{J} \equiv \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix} = r_{ss} \begin{pmatrix} r_{pp}/r_{ss} & r_{ps}/r_{ss} \\ r_{sp}/r_{ss} & 1 \end{pmatrix}$$

isotropic material

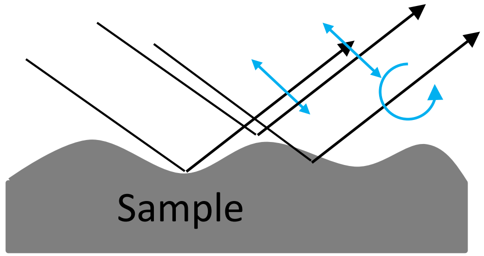
$$= \begin{pmatrix} r_{pp} & 0 \\ 0 & r_{ss} \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2}\left\langle |J_{pp}|^2 + |J_{ss}|^2 + |J_{sp}|^2 + |J_{ps}|^2 \right\rangle & \frac{1}{2}\left\langle |J_{pp}|^2 - |J_{ss}|^2 + |J_{sp}|^2 - |J_{ps}|^2 \right\rangle & \left\langle \mathrm{Re}\left(J_{ps} J^*_{pp} + J_{ss} J^*_{sp}\right) \right\rangle & -\left\langle \mathrm{Im}\left(J_{ps} J^*_{pp} + J_{ss} J^*_{sp}\right) \right\rangle \\ \frac{1}{2}\left\langle |J_{pp}|^2 - |J_{ss}|^2 + |J_{ps}|^2 - |J_{sp}|^2 \right\rangle & \frac{1}{2}\left\langle |J_{ss}|^2 + |J_{pp}|^2 - |J_{sp}|^2 - |J_{ps}|^2 \right\rangle & \left\langle \mathrm{Re}\left(J_{ps} J^*_{pp} - J^*_{sp} J_{ss}\right) \right\rangle & \left\langle \mathrm{Im}\left(-J_{ps} J^*_{pp} + J^*_{sp} J_{ss}\right) \right\rangle \\ \left\langle \mathrm{Re}\left(J^*_{pp} J_{sp} + J^*_{ps} J_{ss}\right) \right\rangle & \left\langle \mathrm{Re}\left(J^*_{pp} J_{sp} - J^*_{ps} J_{ss}\right) \right\rangle & \left\langle \mathrm{Re}\left(J^*_{pp} J_{ss} + J^*_{ps} J_{sp}\right) \right\rangle & \left\langle \mathrm{Im}\left(-J^*_{pp} J_{ss} + J^*_{ps} J_{sp}\right) \right\rangle \\ \left\langle \mathrm{Im}\left(J^*_{pp} J_{sp} + J^*_{ps} J_{ss}\right) \right\rangle & \left\langle \mathrm{Im}\left(J^*_{pp} J_{sp} - J^*_{ps} J_{ss}\right) \right\rangle & \left\langle \mathrm{Im}\left(J^*_{pp} J_{ss} + J^*_{ps} J_{sp}\right) \right\rangle & \left\langle \mathrm{Re}\left(J^*_{pp} J_{ss} - J^*_{ps} J_{sp}\right) \right\rangle \end{pmatrix}$$

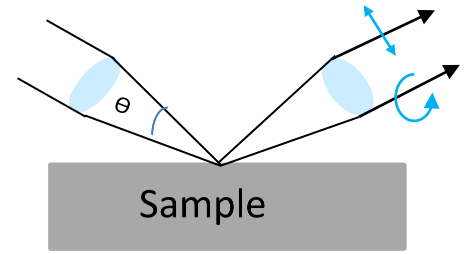
Field amplitudes $E_x$, $E_y$ are defined statistically: The Stokes vector components are linear combinations of the second moments of their joint probability distributions, which are directly related to the experimentally measurable intensities.

# Experimental observations- Depolarization occurs:

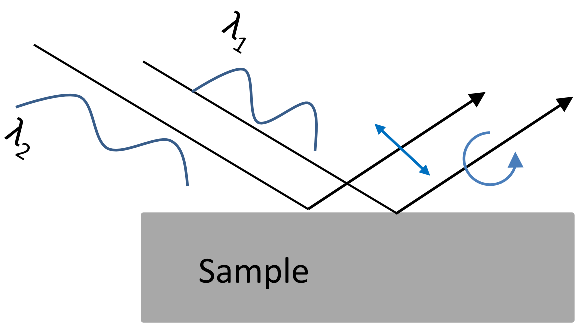
- Surface scattering



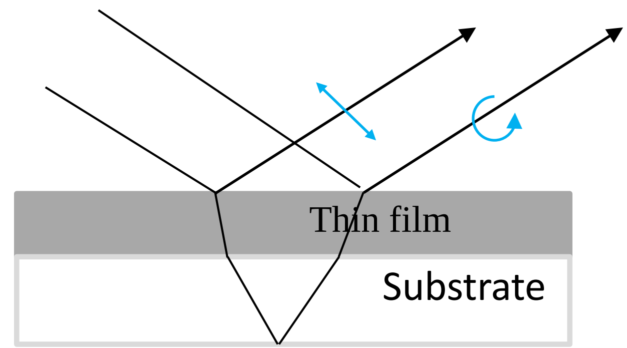

- Incidence angle distribution



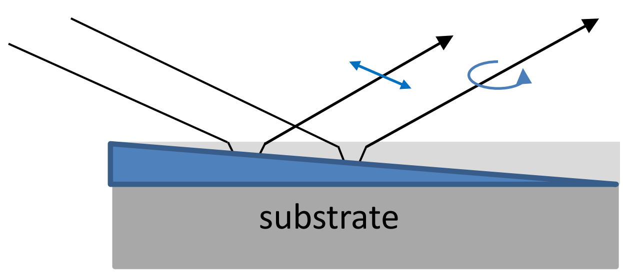

- Wavelength variation

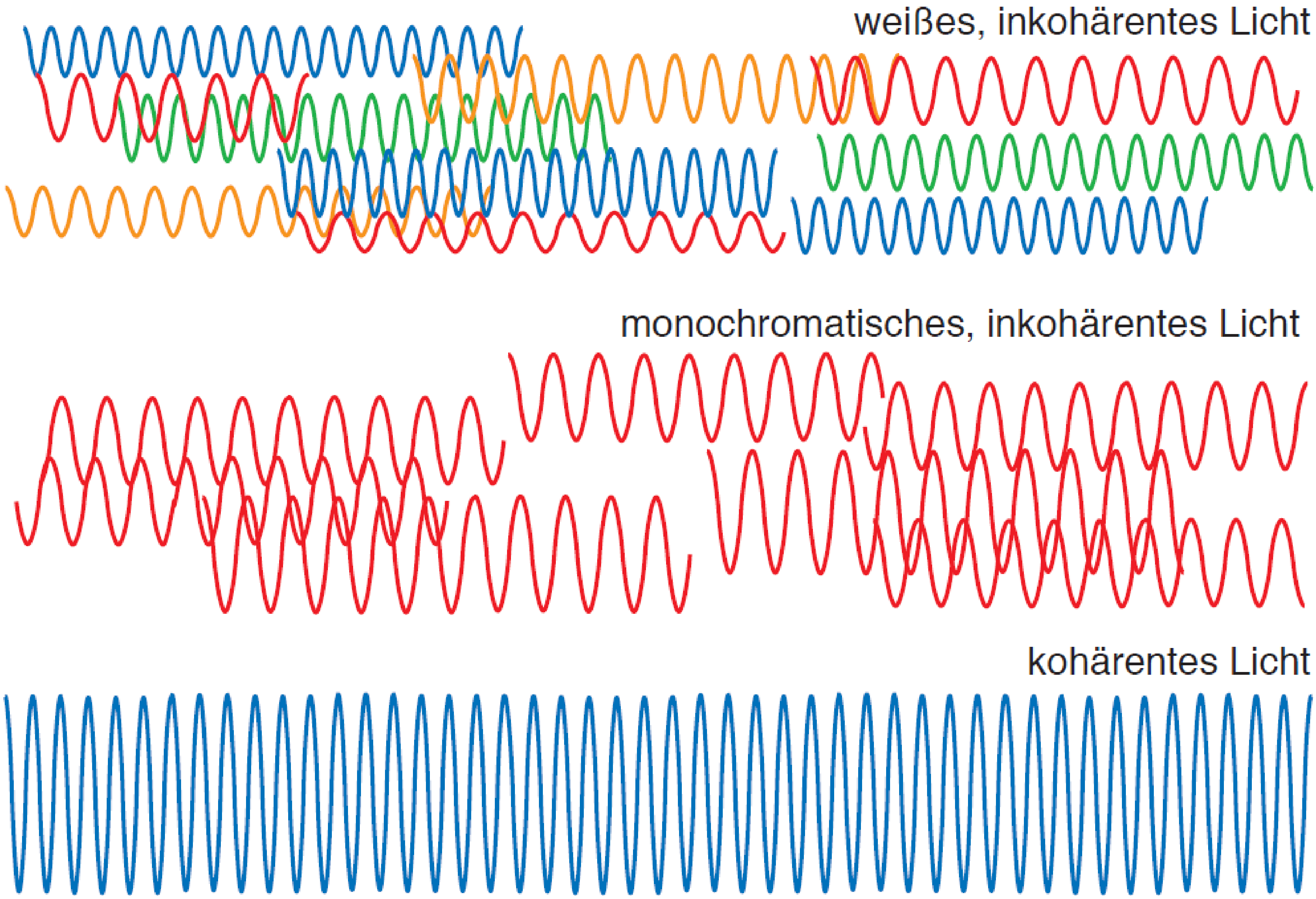



- Backside reflection




- Thickness inhomogeneity




weißes, inkohärentes Licht
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kohärentes Licht

# Decoherence

**We never measure in optics the fields, we always measure 2nd moments:**

$\omega \sim 10^{15} - 10^{16} s^{-1}$; fastest Detector ~ ps; at least $10^6$ oszillations measured

$$I = \underbrace{\varepsilon_0\, n\, c}_{K} \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left( \sum_i \vec{E}_i(t) \right)^2 dt \qquad Example: \quad I = K \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left( \vec{E}_1(t) + \vec{E}_2(t) \right)^2 dt$$

$$\vec{E}_1(\vec{r},t) \triangleq \vec{E}_{01} \cos(\vec{k}_1 \vec{r} - \omega t + \varphi_1) \qquad \vec{E}_2(\vec{r},t) \triangleq \vec{E}_{02} \cos(\vec{k}_2 \vec{r} - \omega t + \varphi_2)$$

$$I = K \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left( \vec{E}_1(t) + \vec{E}_2(t) \right)^2 dt =$$

$$K \left( \frac{1}{2} \vec{E}_{01}^2 + \frac{1}{2} \vec{E}_{02}^2 + \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 2 \vec{E}_{01} \vec{E}_{02} \cos(\vec{k}_1 \vec{r} - \omega t + \varphi_1) \cos(\vec{k}_2 \vec{r} - \omega t + \varphi_2) dt \right) =$$

$$K \left( \frac{1}{2} \vec{E}_{01}^2 + \frac{1}{2} \vec{E}_{02}^2 + \vec{E}_{01} \vec{E}_{02} \cos\left( \left( \vec{k}_1 - \vec{k}_2 \right) \vec{r} + (\varphi_2 - \varphi_1) \right) \right) \underbrace{=}_{\vec{E}_{01} = \vec{E}_{02}}$$

$$K \left( \vec{E}_0^2 + \vec{E}_0^2 \cos\left( \left( \vec{k}_1 - \vec{k}_2 \right) \vec{r} + (\varphi_2 - \varphi_1) \right) \right)$$

**If phase is not well defined => Statistical Optics!**

# Temporal and Spatial Decoherence

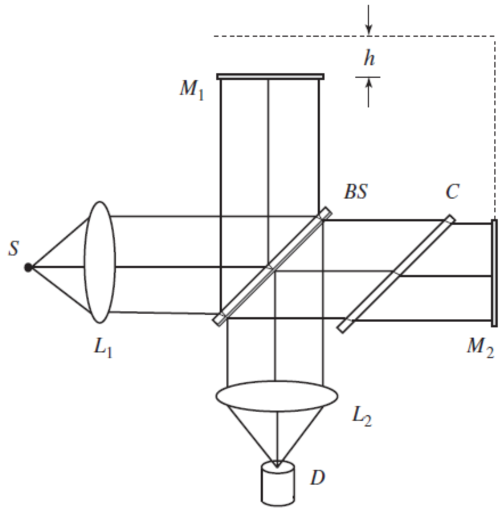



**Figure 5.1** The Michelson interferometer, including the source $S$, the lenses $L_1$ and $L_2$, mirrors $M_1$ and $M_2$, beam splitter $BS$, compensator $C$, and detector $D$.

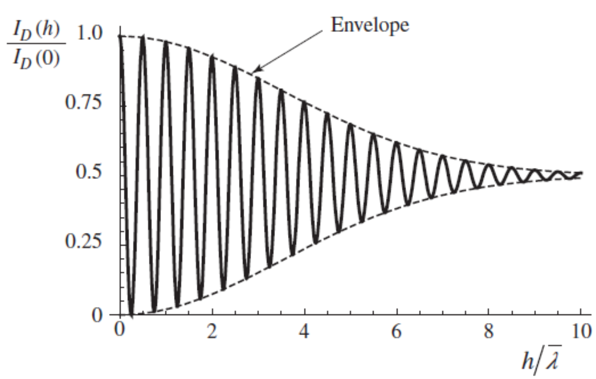



**Figure 5.2** Normalized intensity incident on detector $D$ vs. normalized mirror displacement $h/\bar{\lambda}$. The spectrum shape has been assumed to be Gaussian, centered at $\bar{\lambda}$, for this plot. The envelope of the fringe pattern is drawn dotted.

$$I_D(h) = \left\langle \left| K_1 \mathbf{u}(t) + K_2 \mathbf{u}\left(t + \frac{2h}{c}\right) \right|^2 \right\rangle ,$$

$$I_D(h) = K_1^2 \left\langle |\mathbf{u}(t)|^2 \right\rangle + K_2^2 \left\langle \left| \mathbf{u}\left(t + \frac{2h}{c}\right) \right|^2 \right\rangle$$

$$+ K_1 K_2 \left\langle \mathbf{u}\left(t + \frac{2h}{c}\right) \mathbf{u}^*(t) \right\rangle + K_1 K_2 \left\langle \mathbf{u}^*\left(t + \frac{2h}{c}\right) \mathbf{u}(t) \right\rangle .$$

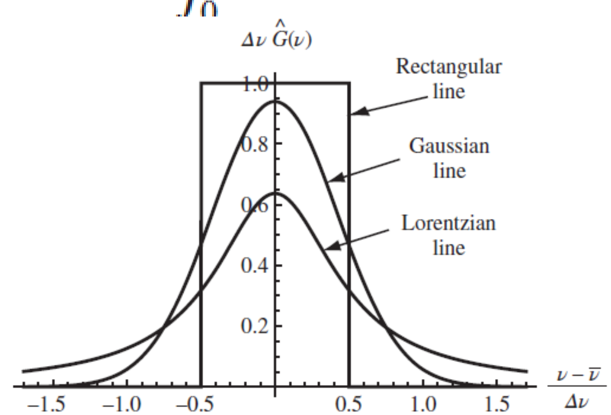
$$\boldsymbol{\Gamma}(\tau) = \langle \mathbf{u}(t+\tau)\, \mathbf{u}^*(t) \rangle .$$

$$I_D(h) = \left(K_1^2 + K_2^2\right) I_0 + K_1 K_2 \boldsymbol{\Gamma}\left(\frac{2h}{c}\right) + K_1 K_2 \boldsymbol{\Gamma}^*\left(\frac{2h}{c}\right)$$

$$= \left(K_1^2 + K_2^2\right) I_0 + 2 K_1 K_2 \,\mathrm{Re}\left\{ \boldsymbol{\Gamma}\left(\frac{2h}{c}\right) \right\} ,$$

**via Wiener Khitchin theorem (FT)**

$$\boldsymbol{\Gamma}(\tau) = \int_0^\infty 4\, \mathcal{G}^{(r,r)}(\nu) e^{-j2\pi\nu\tau}\, d\nu ,$$



# Temporal and **Spatial** Decoherence (quasimonochromatic)

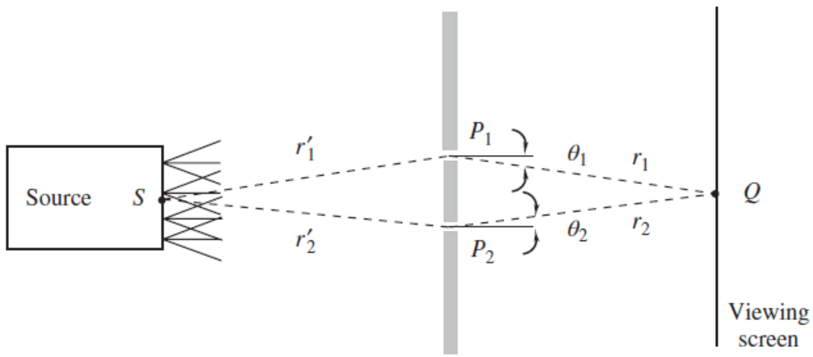



**Figure 5.12** Young's interference experiment.

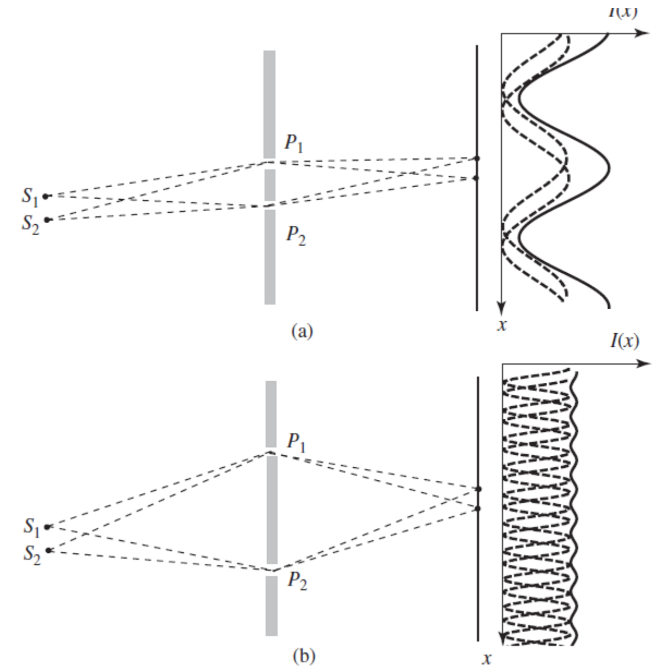
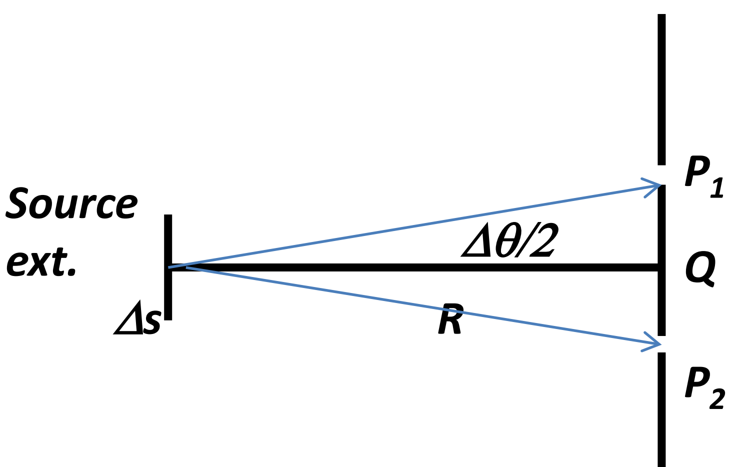
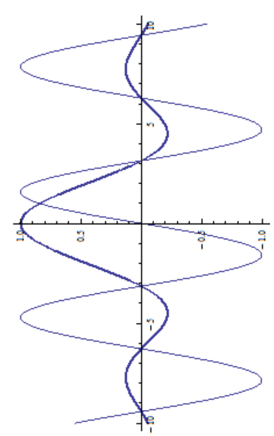



**Figure 5.13** Physical explanation for loss of fringe visibility at large pinhole spacings: (a) small pinhole spacing and (b) large pinhole spacing.

$$\vec{E}_0^{\,2} + \vec{E}_0^{\,2} \cos((\vec{k}_1 - \vec{k}_2)\vec{r} + (\varphi_2 - \varphi_1))$$

$$\mathbf{u}(Q,t) = \mathbf{K}_1\mathbf{u}\left(P_1, t - \frac{r_1}{c}\right) + \mathbf{K}_2\mathbf{u}\left(P_2, t - \frac{r_2}{c}\right),$$



$$Q^2 = \Delta A = \frac{R^2 \overline{\lambda}^2}{(\Delta s)^2} \sim (50\mu m)^2$$

Can be proven: van Cittert – Zernike Theorem

Experiments:

Sun: 0.1 mm^2

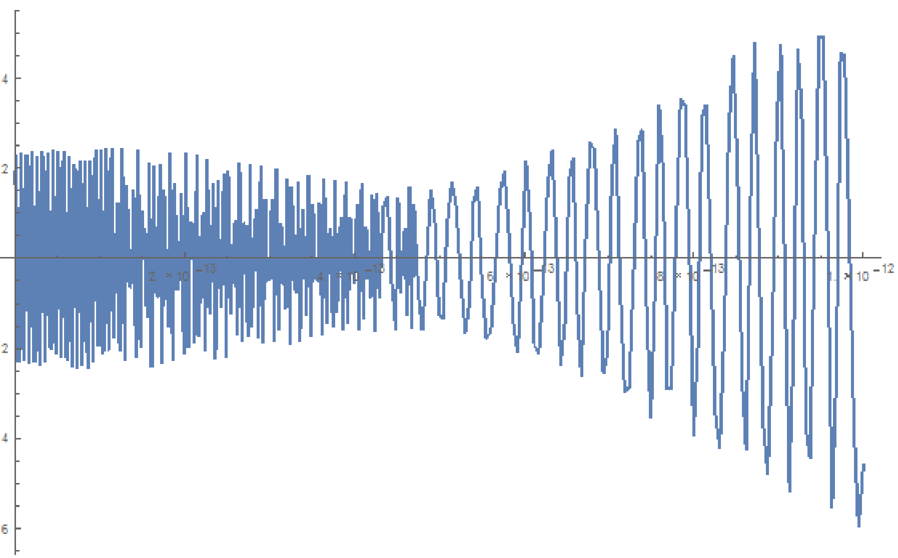
Orion star: 6 m^2

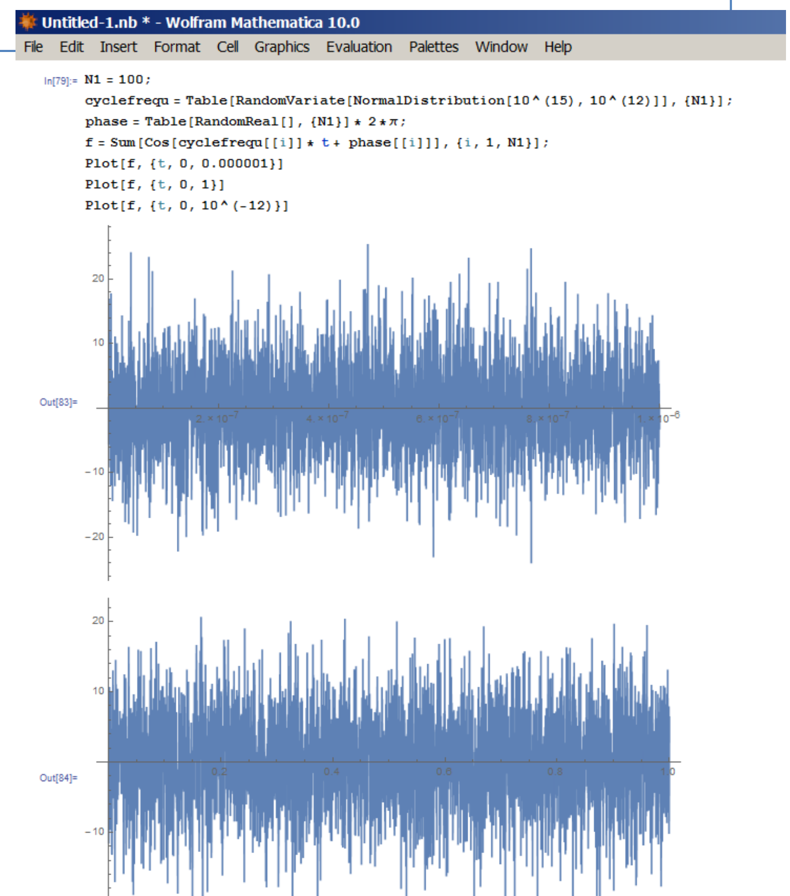
# *2 Photons (2 interfering beams) => N photons (N interfering beams)*

- Finite coherence length of your light source $$l_c = \frac{\bar{\lambda}}{\Delta\lambda}\bar{\lambda} \leq 100\mu m$$
- Monochromator resolution ~

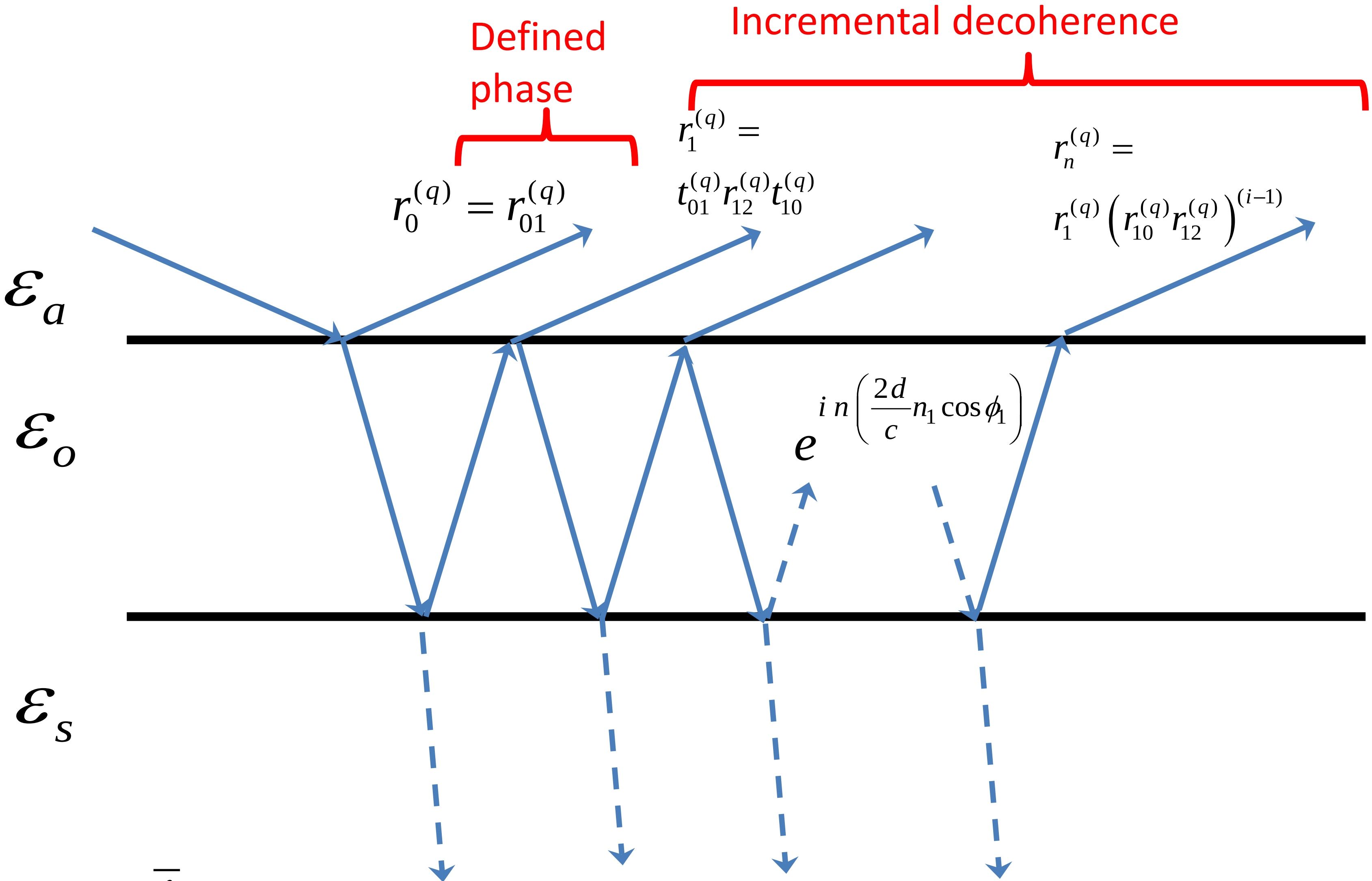
$$\Delta\lambda \sim 3nm -> \Delta\omega \sim 10^{13} s^{-1}$$

- 1 Watt ~ $10^{19}$ photons/s, they all interfere, rather complicated beating pattern („purely?“ statistical?): N=100, 1 ps, 1 µs, 1 s



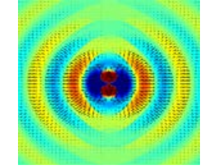


# Spectroscopic ellipsometry for a thick overlayer:

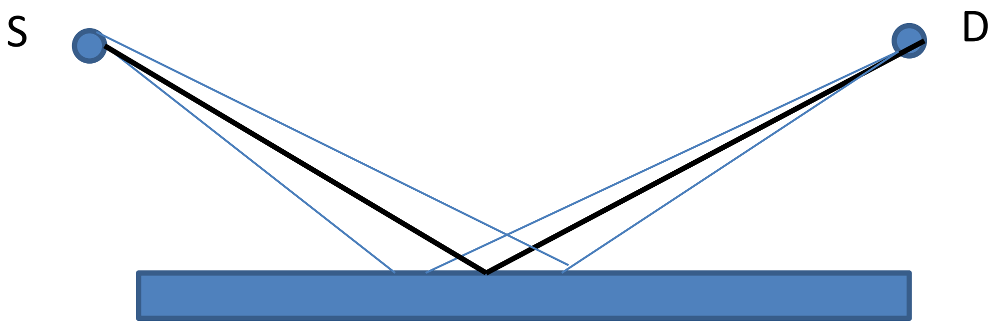



$l_c = \frac{\bar{\lambda}}{\Delta\lambda}\bar{\lambda} \sim \frac{500nm}{3nm} 500nm \leq 100\mu m$ in Air! ( refractive index goes in, too)

# Decoherence

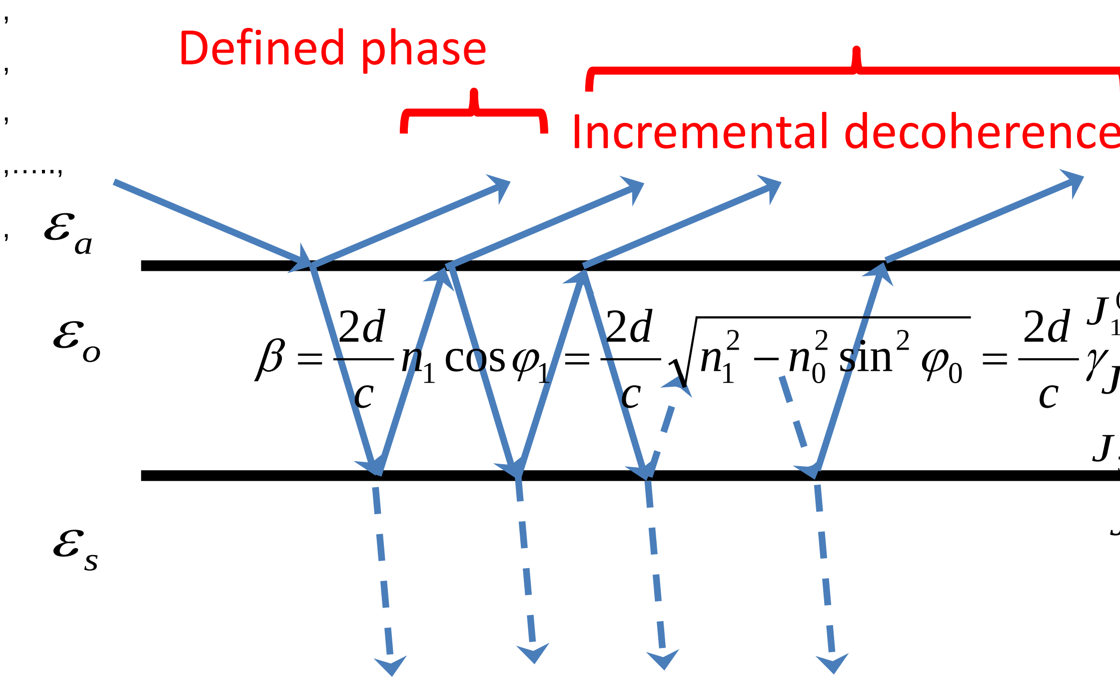


$$2\frac{1}{2}\vec{E}_0^{\,2} + \vec{E}_0^{\,2}\cos(\vec{k}_1\vec{r} - \vec{k}_2\vec{r} + (\varphi_2 - \varphi_1))$$




- Lets assume we have the most perfect sample in the world, with a „most“ homogeneous thickness: **$SiO_2$ on Si**
- Finite coherence length of your light source $l_c = \frac{\bar{\lambda}}{\Delta\lambda}\bar{\lambda} \leq 100\mu m$
- Monochromator resolution ~ $\Delta\lambda \sim 3nm -> \Delta\omega \sim 10^{13} s^{-1}$
- 1 Watt ~ $10^{19}$ photons/s, they all interfere, rather complicated beating pattern („purely?“ statistical?)
- If $\varphi_2$-$\varphi_1$ is **purely stochastic** $[-\pi, \pi[$, the measured intensity is just the sum of the intensities, **all coherence lost!**

# Retardation: Ewald Oseen




$$\beta = \frac{2d}{c} n_1 \cos\varphi_1 = \frac{2d}{c}\sqrt{n_1^2 - n_0^2 \sin^2\varphi_0} = \frac{2d}{c}\gamma$$

$$E_{rad} \propto \ddot{p}(t - \frac{r}{c})$$

$$J_0^{(q)} r_0^{(q)} = r_{01}^{(q)}$$

$$J_1^{(q)} = r_1^{(q)} = t_{01}^{(q)} r_{12}^{(q)} t_{10}^{(q)} = r_{12}^{(q)}\left(1 - r_{01}^{(q)2}\right)$$

$$J_2^{(q)} = r_2^{(q)} = t_{01}^{(q)} r_{12}^{(q)} r_{10}^{(q)} r_{12}^{(q)} t_{10}^{(q)} = r_1^{(q)}\left(r_{10}^{(q)} r_{12}^{(q)}\right.$$

$$J_3^{(q)} = r_3^{(q)} = r_2^{(q)}\left(r_{10}^{(q)} r_{12}^{(q)}\right) = r_1^{(q)}\left(r_{10}^{(q)} r_{12}^{(q)}\right)^2$$

$$J_i^{(s,p)} = r_i^{(q)} = r_1^{(q)}\left(r_{10}^{(q)} r_{12}^{(q)}\right)^{(i-1)}$$

## Correlation functions in time

--→ --→---→----→---------→

$$I_D^{(s,p)} = \left\langle \left| r_0 E(t) + r_1 E(t+\beta) + r_2 E(t+2\beta) + \dots + r_n E(t+n\beta) \right|^2 \right\rangle =$$

$$\left\langle \left| r_0 E(t) \right|^2 \right\rangle + \left\langle \left| r_1 E(t+\beta) \right|^2 \right\rangle + \dots \left\langle \left| r_n E(t+n\beta) \right|^2 \right\rangle + \sum_{n=1}^{\infty} \sum_{\substack{m=0 \\ m<n}}^{\infty} \left\langle \left| r_m E(t+m\beta) r_n E(t+n\beta) \right|^2 \right\rangle =$$

$$\sum_{m=0}^{\infty} \left| r_m \right|^2 \left\langle E(t) r_n E(t) \right\rangle + \sum_{n=1}^{\infty} \sum_{\substack{m=0 \\ m<n}}^{\infty} \left| r_m r_n \right| \left\langle \left| E(t) E(t + (m-n)\beta) \right|^2 \right\rangle$$

’Correlation functions in time **via Wiener Khitchin theorem (FT)**

’ --→ --→---→----→---------→

’......,

**Convolution in frequency space**

$$\left\langle J_k J_m^* \right\rangle_t (\omega;\Delta\omega) = \int_{-\infty}^{\infty} J_k(\omega') J_m^*(\omega') w(\omega'-\omega;\Delta\omega)\, d\omega'$$

Assuming e.g. a monochromator resolution function (Gaussian, Lorentz, equal distribution) puts us in the position to evaluate each term and sum it:

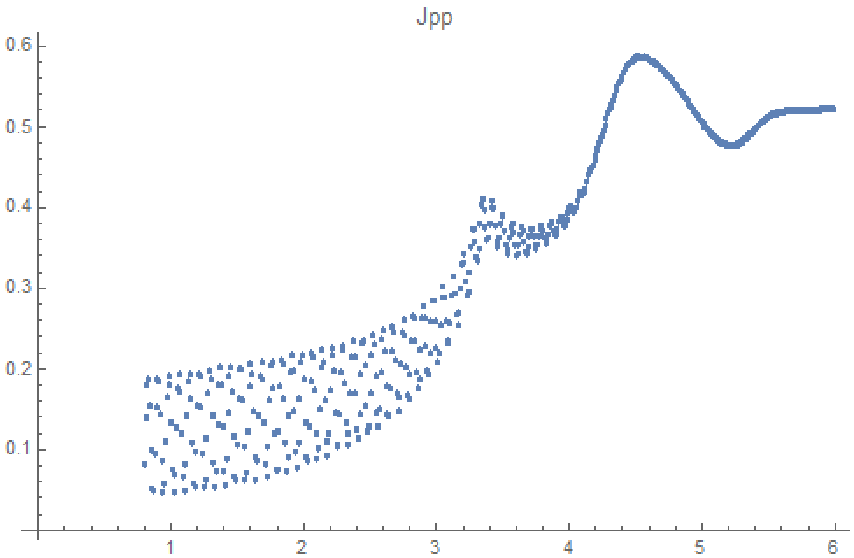
$$A_{nm}^{(qq')} \approx r_n^{(q)} r_m^{(q')*} \exp\left[-\omega(n+m)\operatorname{Im}\beta\right] \left\langle \exp\left[i\omega(n-m)\operatorname{Re}\beta\right] \right\rangle =$$

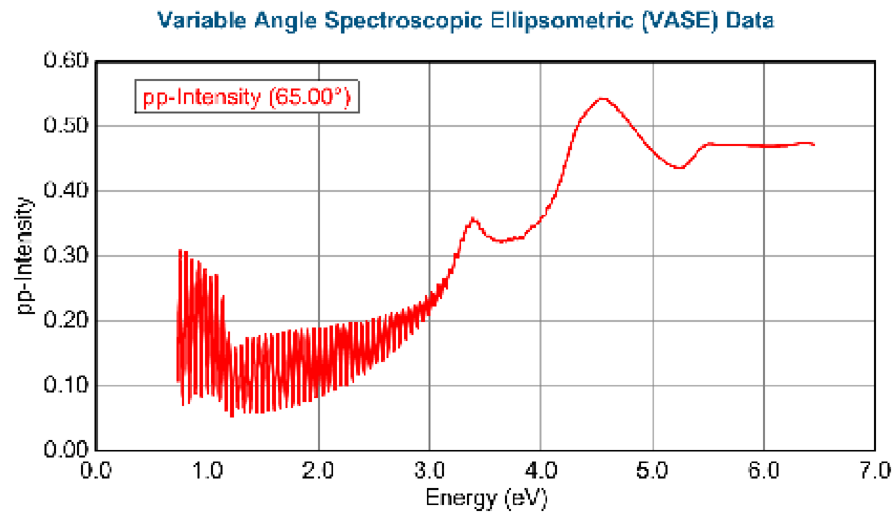
$$= r_n^{(q)} r_m^{(q')*} \exp\left[-\omega(n+m)\operatorname{Im}\beta\right] \frac{1}{\sqrt{2\pi}\Delta\omega} \int_{-\infty}^{\infty} \exp\left[-\frac{(\omega-\omega')^2}{2\Delta\omega^2}\right] \exp\left[i\omega'(n-m)\operatorname{Re}\beta\right] d\omega' =$$

$$= r_n^{(q)} r_m^{(q')*} \exp\left[-\omega(n+m)\operatorname{Im}\beta\right] \exp\left[i\omega(n-m)\operatorname{Re}\beta\right] \boxed{\exp\left[-(n-m)^2 (\operatorname{Re}[\beta])^2 \frac{\Delta\omega^2}{2}\right]} =$$

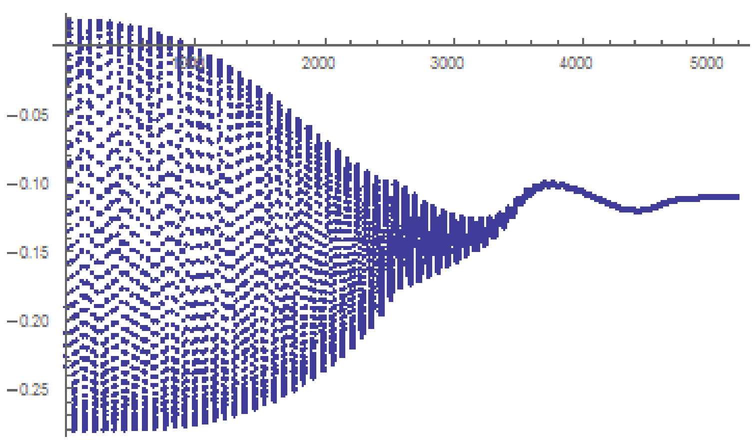
$$= A_n^{(q)} A_m^{(q')*} \exp\left[-(n-m)^2 \operatorname{Re}^2\beta \frac{\Delta\omega^2}{2}\right] = A_n^{(q)} A_m^{(q')*} f_{nm}^G(\Delta\omega)$$

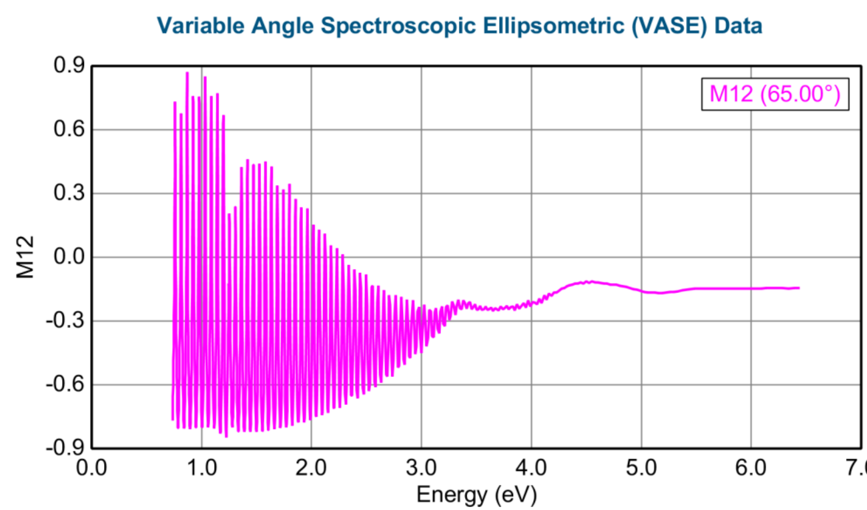
# Forward Simulation (all 16 MMs) of **10µmSiO/Si** ( 3nm spectral width)



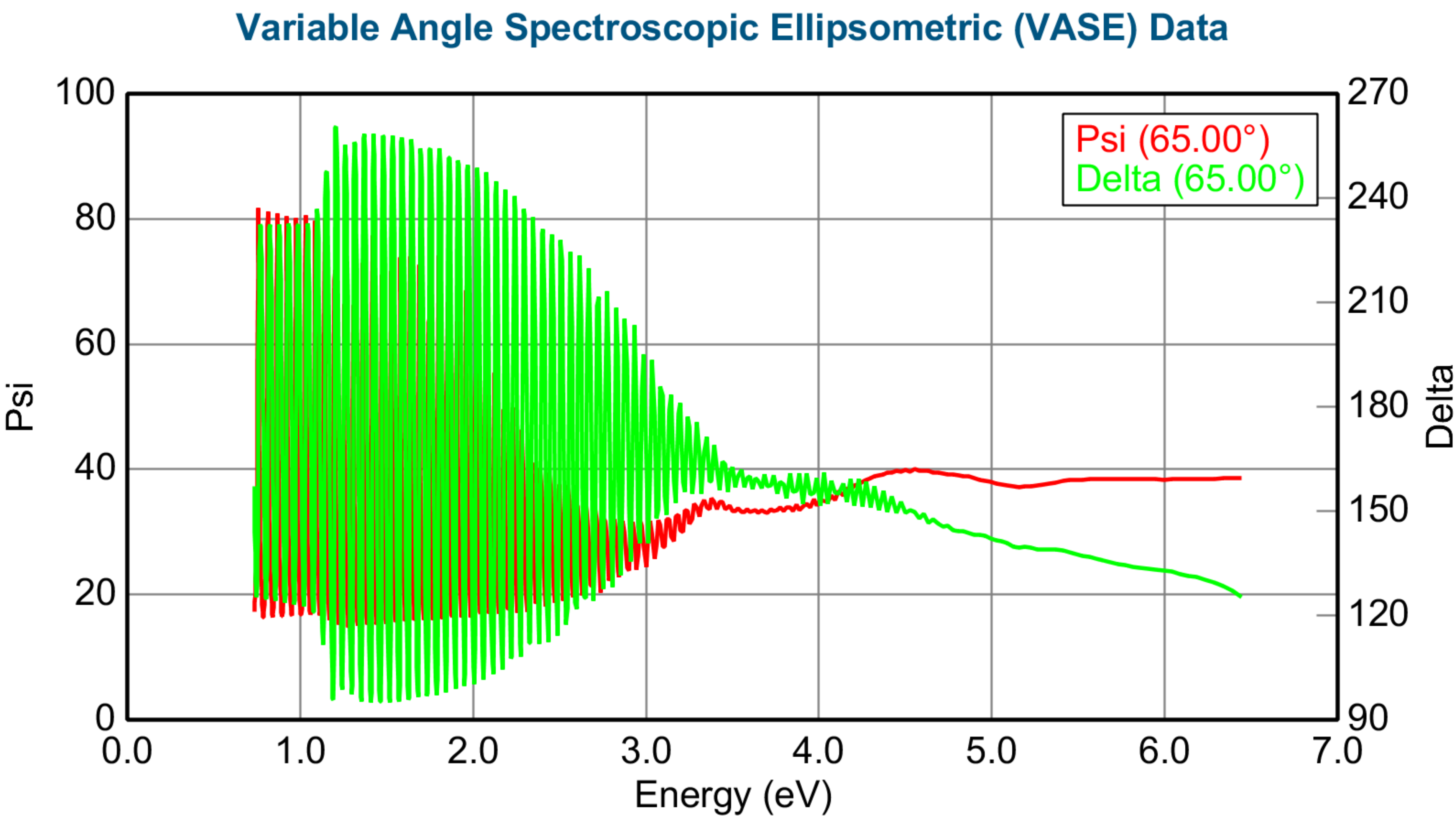




M12

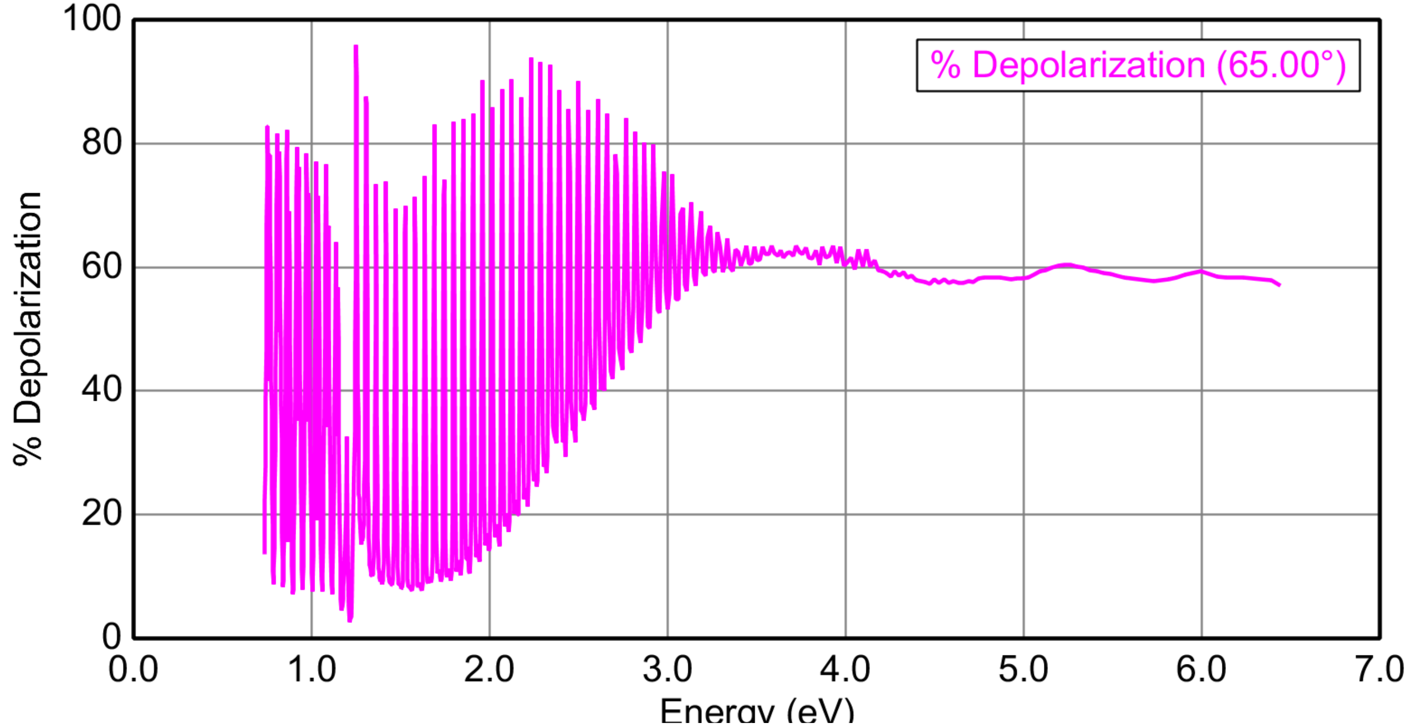


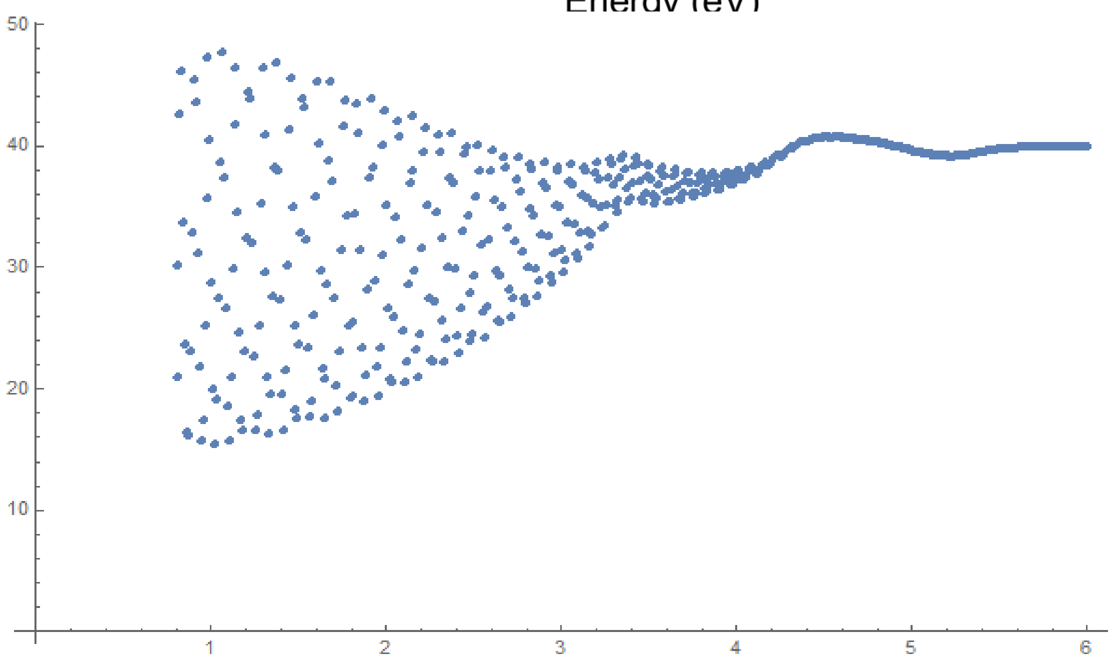



10 µm $SiO_2$ on Si



# Variable Angle Spectroscopic Ellipsometric (VASE) Data



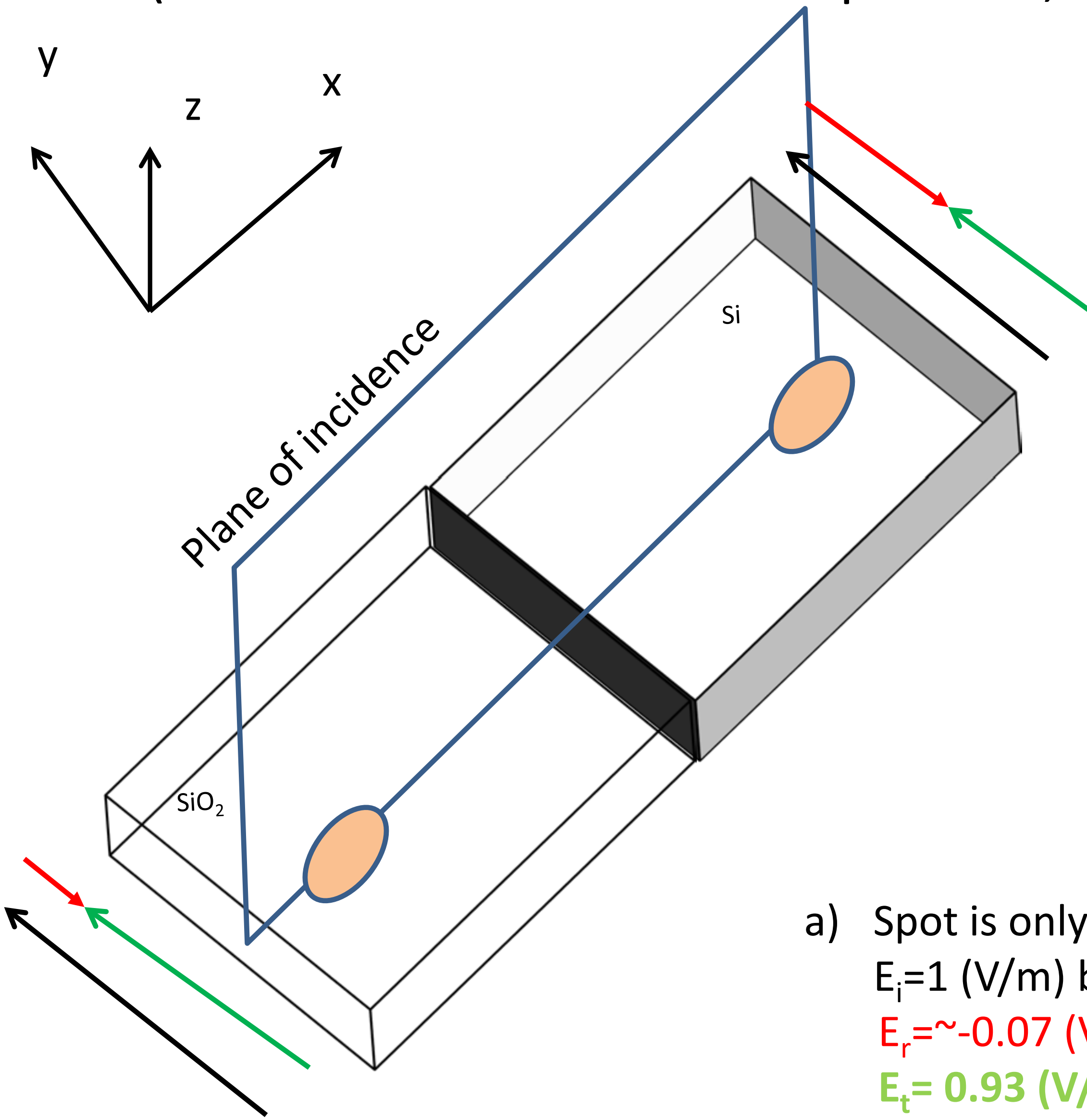

Normalized M(1,1)
Sample assumed as perfectly homogeneous

# Crosspolarization and Depolarization in Ellipsometry at Inner Boundaries

- Introduction and Motivation
- Decoherence
- Depolarization
  - Temporal
- ***Continuity Conditions at Inner Boundaries***
- Crosspolarization
- **Depolarization**
  - **Spatial**

# Crosspolarization

(instead of mathematics => pictorial, normal incidence)




Lets start with s-polarized light

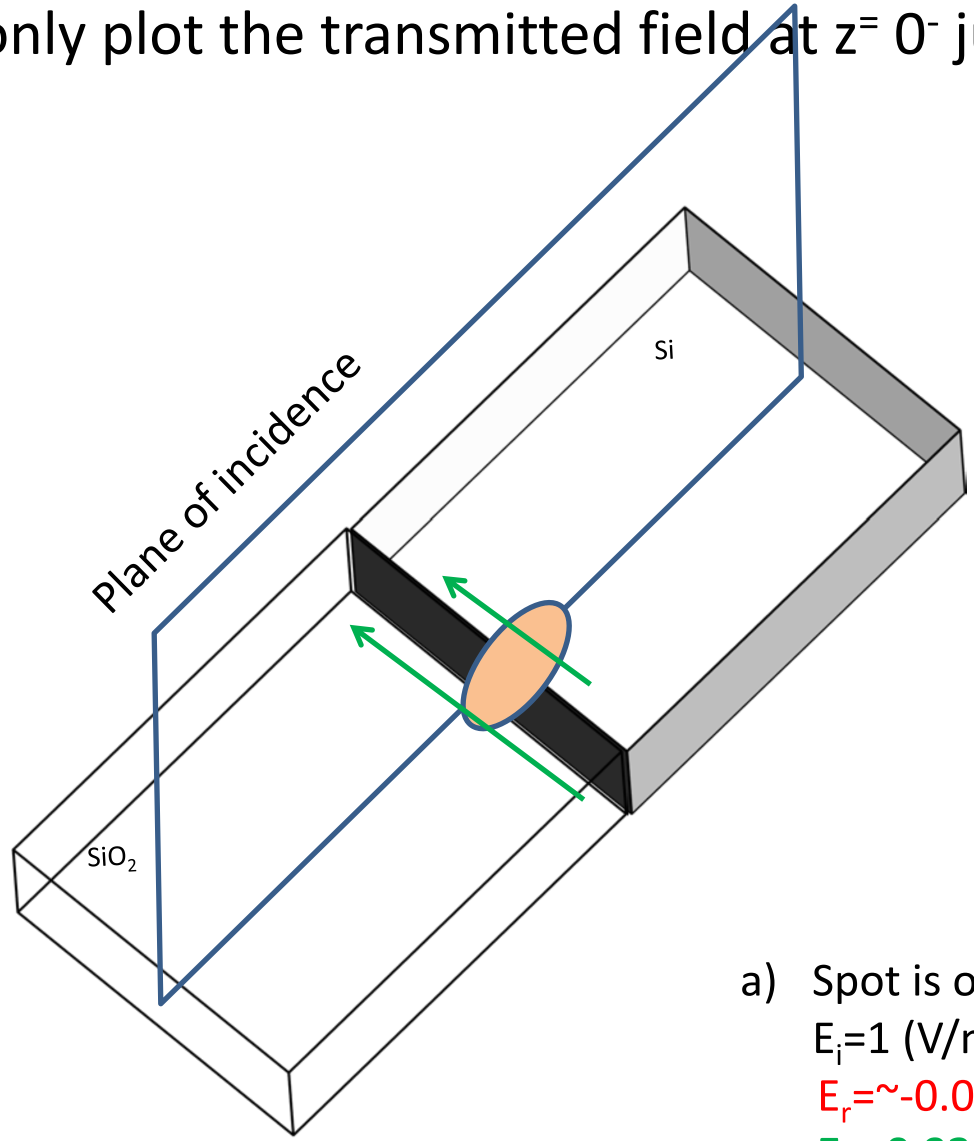
$$\hbar\omega \sim 1eV$$

a) Spot is only on the Si
   $E_i$=1 (V/m) black arrow
   $E_r$=~-0.5 (V/m) red arrow
   **$E_t$= 0.5 (V/m) green arrow**

a) Spot is only on the SiO2
   $E_i$=1 (V/m) black arrow
   $E_r$=~-0.07 (V/m) red arrow
   **$E_t$= 0.93 (V/m) green arrow**

# Crosspolarization

(now lets move the spot towards the interface and only plot the transmitted field at $z = 0^-$ just inside the material)

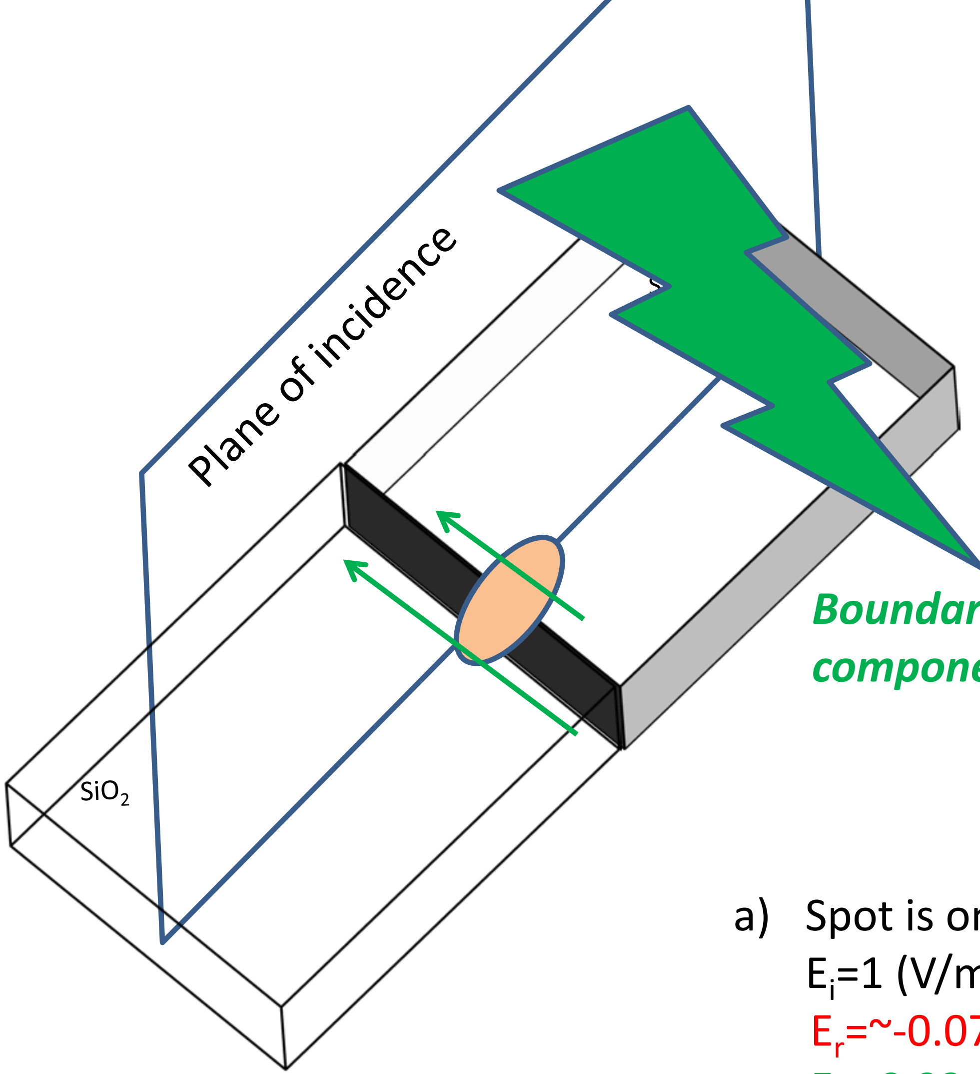



a) Spot is only on the Si
$E_i$=1 (V/m) black arrow
$E_r$=~-0.5 (V/m) red arrow
**$E_t$= 0.5 (V/m) green arrow**

a) Spot is only on the SiO2
$E_i$=1 (V/m) black arrow
$E_r$=~-0.07 (V/m) red arrow
**$E_t$= 0.93 (V/m) green arrow**

# Crosspolarization

(now lets move the spot towards the interface and only plot the transmitted field at $z=0^+$- just inside the material)




a) Spot is only on the Si
$E_i$=1 (V/m) black arrow
$E_r$=~-0.5 (V/m) red arrow
$E_t$= 0.5 (V/m) green arrow

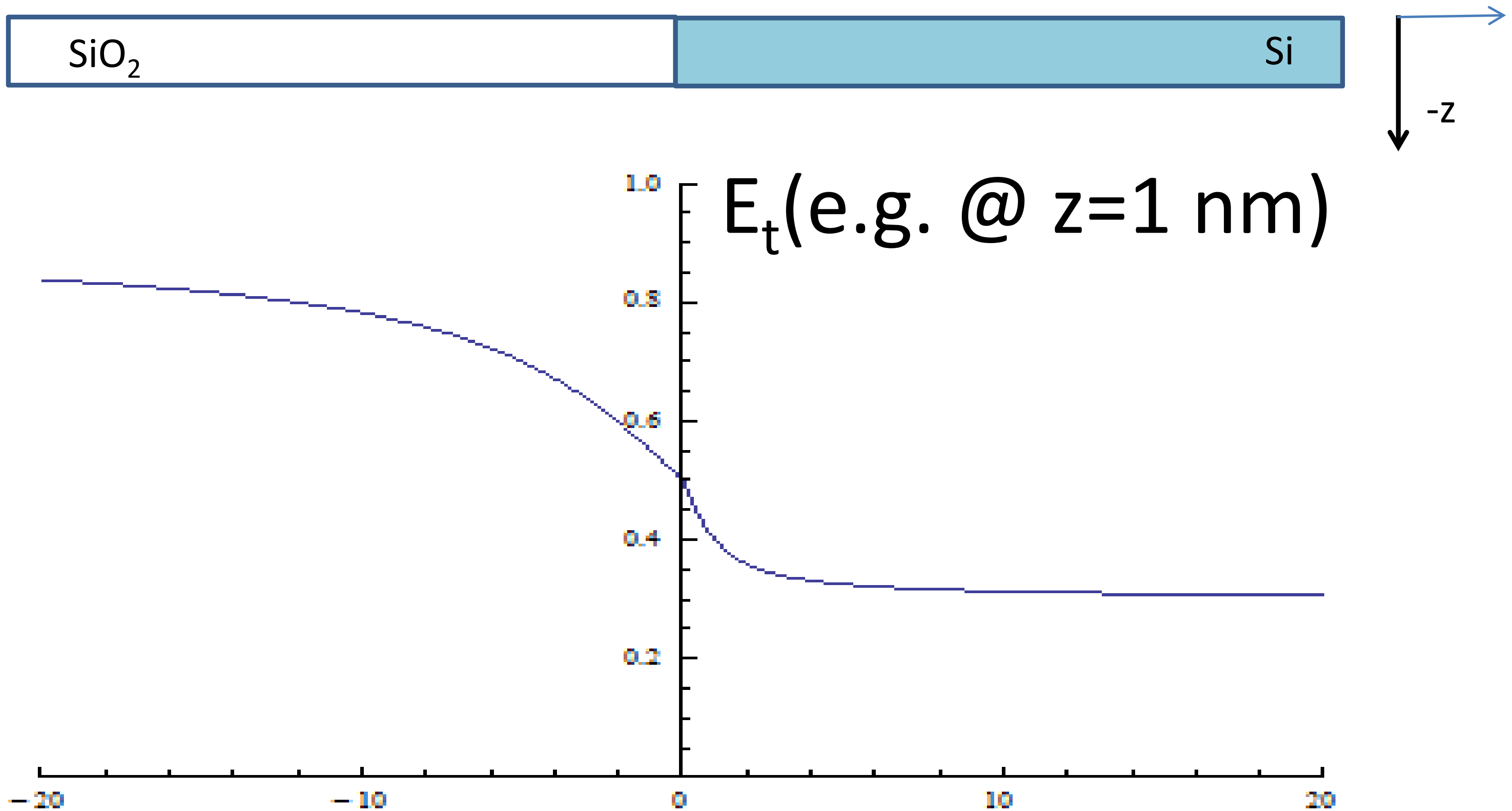
***Boundary conditions on tangential component of the E-field NOT fulfilled***

a) Spot is only on the SiO2
$E_i$=1 (V/m) black arrow
$E_r$=~-0.07 (V/m) red arrow
$E_t$= 0.93 (V/m) green arrow

# Q: How does nature solve this problem?

# How does nature solve this problem?

A: Maxwell equations require EVANESCENT FIELDS (at each depth, i.e. for any z)



# Numerical Example I

RCWA calculation, (Fresnel), normal incidence (Prof. Lalannes Reticolo Code)

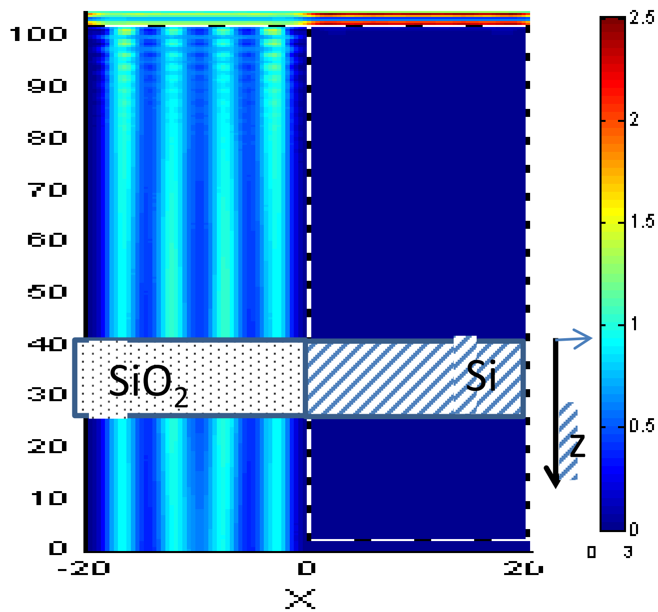
- Pure Si: r=- 0.5558 (n_Si=3.5+ 0.01i)
- Pure $SiO_2$: r=- 0.1667
- ½ Si: ½ $SiO_2$ (in a 40µm periodic cell): -0.3654

*this is NOT equal to arithmetical mean (-0.36125)*

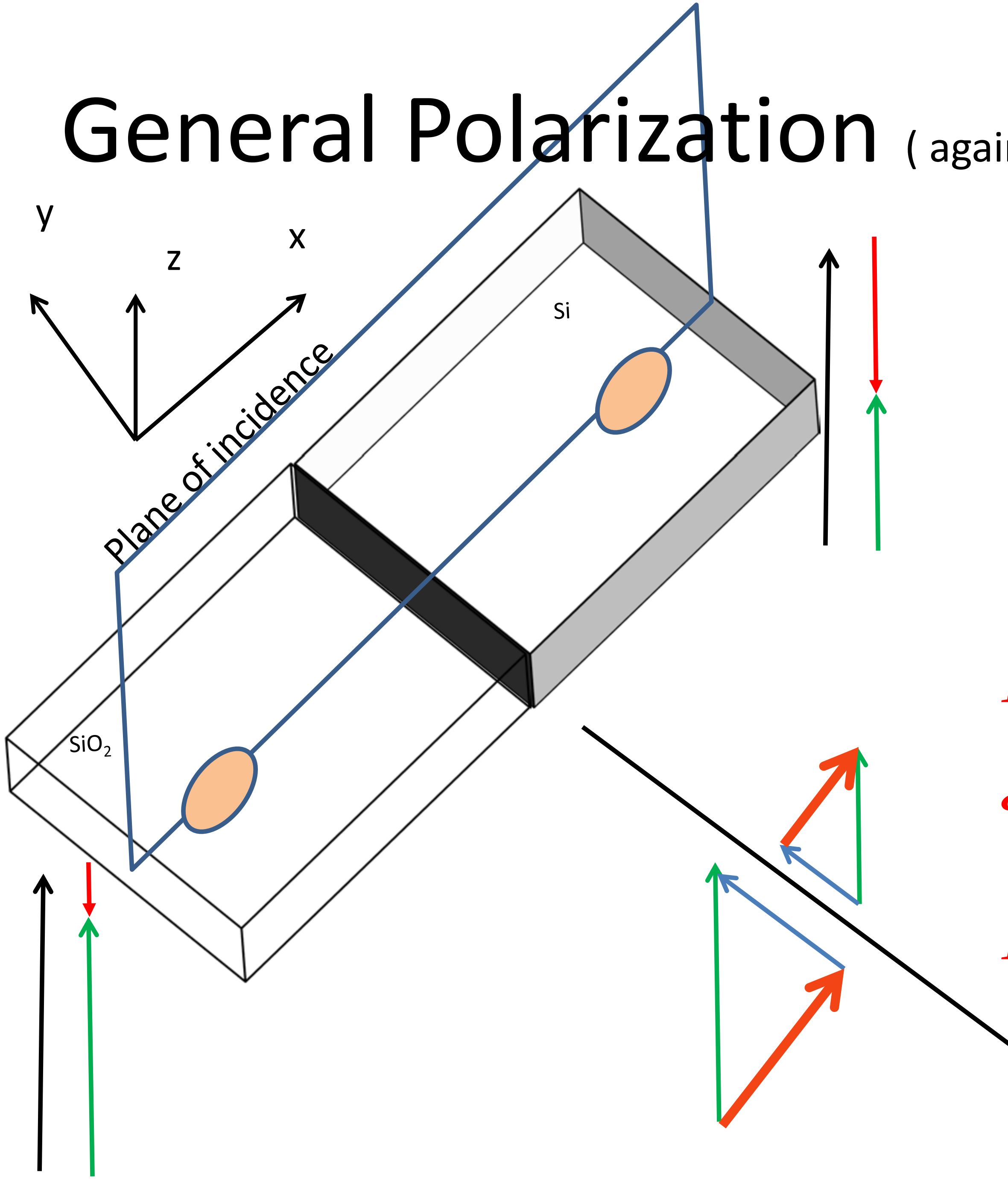
Findings: just s-polarization

- *reflectivities are not linear mean*
- *Both reflected s- waves have different amplitudes*
- *and in case of absorption different phases*
- *Strongly dependent on geometry (periode, …)*
- *Looks in a periodic structure in the depth more like a „Waveguide" mode*

$$\left|Abs(E_y(x, y=0, z)\right|^2$$



# General Polarization ( again s- and p-, normal incidence)



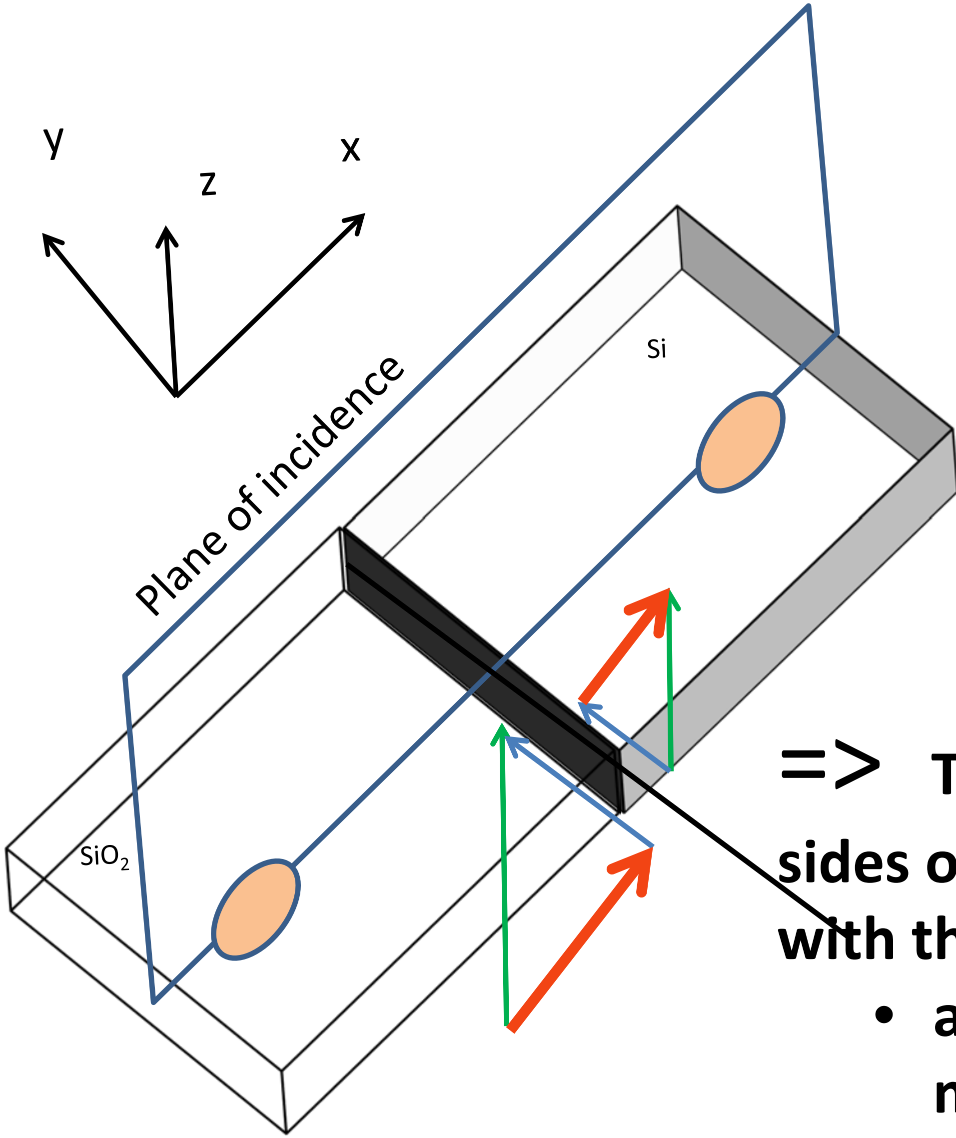

**Red arrows** mark the component of the inconming field normal to the inner boundary.

For the normal component a different boundary condition is valid:

$$D_\perp^{(1)} = D_\perp^{(2)}$$

$$\varepsilon_0\ \varepsilon^{(1)} E_\perp^{(1)} = \varepsilon_0\ \varepsilon^{(2)} E_\perp^{(2)}$$

$$E_\perp^{(1)} = \frac{\varepsilon^{(2)}}{\varepsilon^{(1)}} E_\perp^{(2)}$$




For the normal component a different boundary condition is valid:

$$E_{\perp}^{(1)} = \frac{\varepsilon^{(2)}}{\varepsilon^{(1)}} E_{\perp}^{(2)}$$

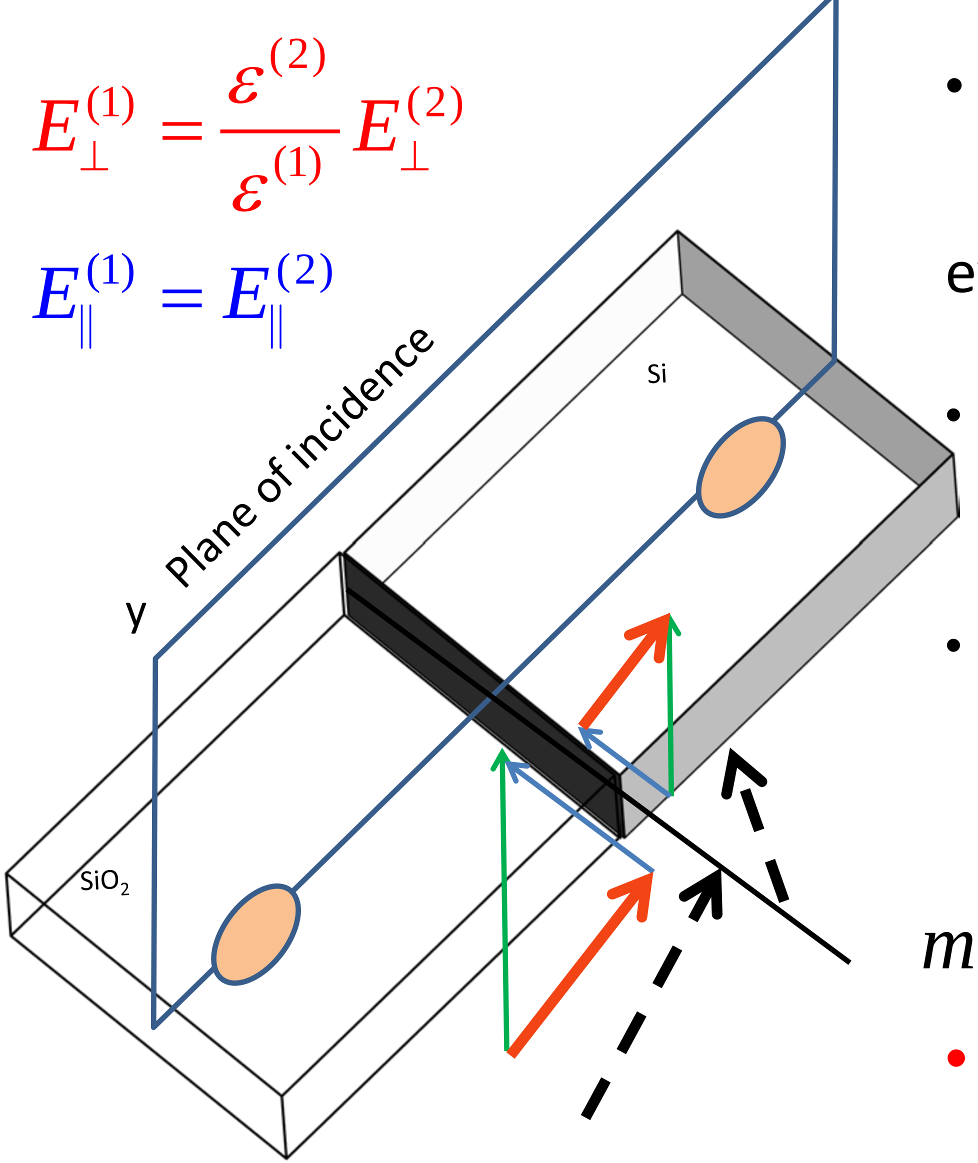
$$E_{\parallel}^{(1)} = E_{\parallel}^{(2)}$$

=> **The transmitted fields on both sides of the boundary in the region with the evanescent fields**

- **are not mutually parallel any more,**
- **are neither parallel to the fields away from the boundary**
- resp. the incident one)

$$E_{\perp}^{(1)} = \frac{\varepsilon^{(2)}}{\varepsilon^{(1)}} E_{\perp}^{(2)}$$

$$E_{\|}^{(1)} = E_{\|}^{(2)}$$




- **Dashed Black Arrows** indicate a coarse approximation for the inner transmitted field (macroscopic field)

even in **ISOTROPIC** materials

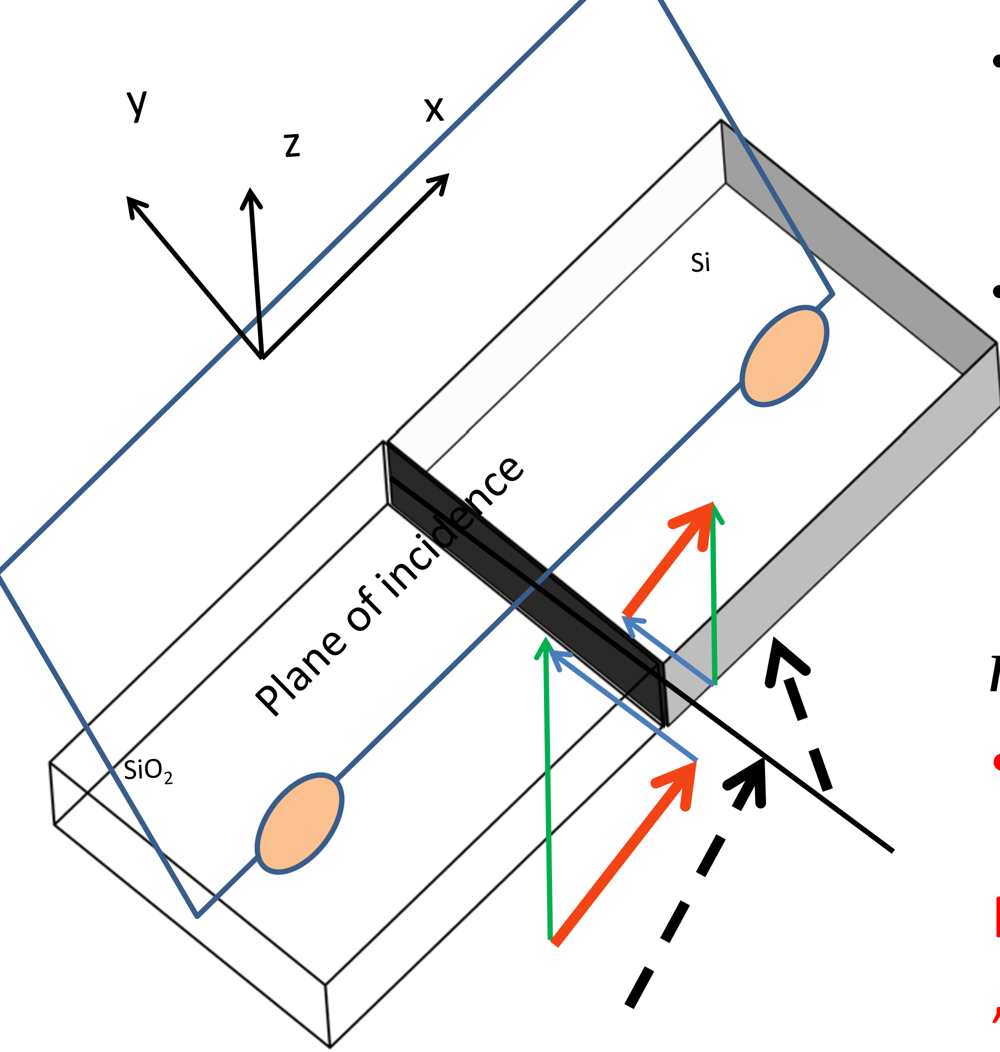
- Field changes direction at the inner boundaries, in metals the normal component even reverses.

- this is the macroscopically correct one, which drives the radiating dipoles close to the boundary (in the evanescent region) in other directions than the incident field.

$$m\,\ddot{\vec{x}} + m\,\gamma\,\dot{\vec{x}} + m\,\omega_0^2\,\vec{x} = q\vec{E}\,e^{-i\omega t}$$

- „Source of **cross-polarization“,**

- often wrongly described as„effective anisotropic “ material (geometry dependent)




- Field changes direction at the inner boundaries, in metals the normal component even reverses.
- this is the macroscopically correct one (perhaps corrected by LFE), which drives the radiating dipoles close to the boundary (in the evanescent region) in other directions than the incident field.

$$m\ddot{\vec{x}} + m\gamma\dot{\vec{x}} + m\omega_0^2\vec{x} = q\vec{E}\,\mathrm{e}^{-i\omega t}$$

- „Source of **cross-polarization**",

Maxwell equation take care of this „equ. of motion" automatically:

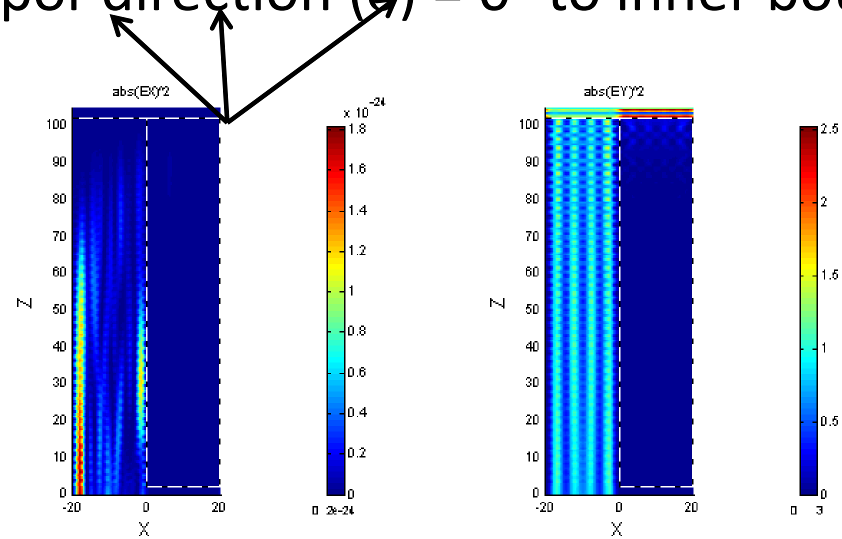
Despite the homogeneous Helmholtz equations SEEM to be decoupled in the single field components, they are coupled via „boundaries":

$$\Delta\vec{E}(\vec{r},t) + \varepsilon(\omega)\frac{\partial^2\vec{E}(\vec{r},t)}{c^2\partial t^2} = 0 \quad \Delta E_x(\vec{r},t) + \varepsilon(\omega)\frac{\partial^2 E_x(\vec{r},t)}{c^2\partial t^2} = 0 \quad \Delta E_y(\vec{r},t) + \varepsilon(\omega)\frac{\partial^2 E_y(\vec{r},t)}{c^2\partial t^2} = 0$$

# Numerical Example II

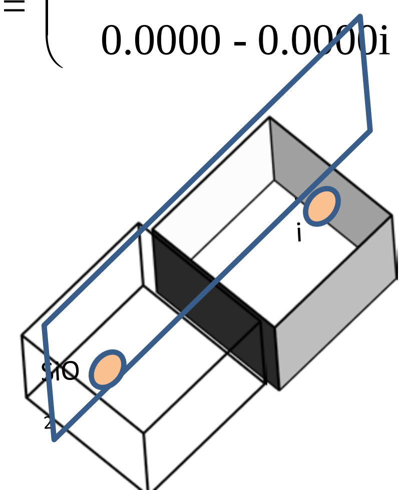
RCWA calculation, normal incidence (θ_in=0) (Si, SiO2),

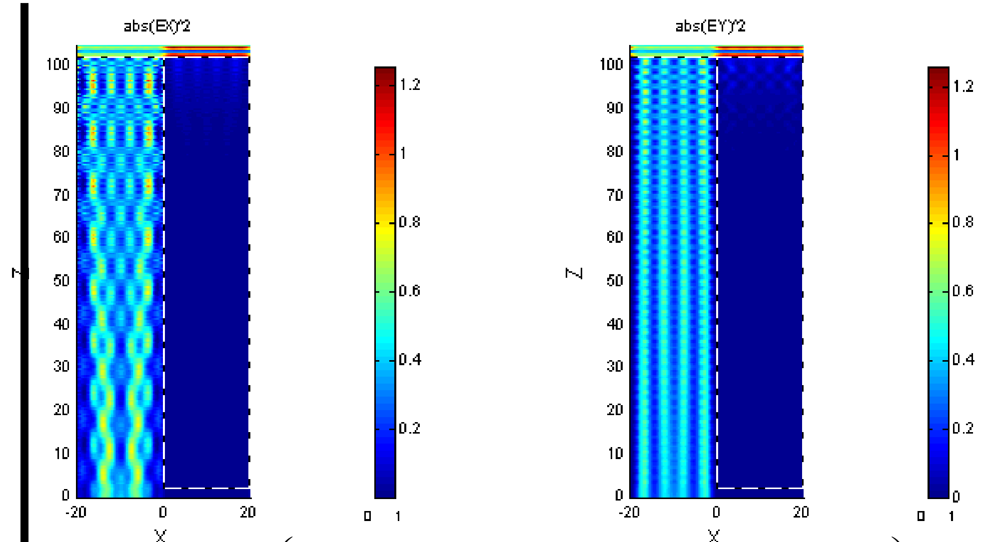
pol-direction (δ) = 0° to inner boundary, pol-direction (δ) = +45° to IB



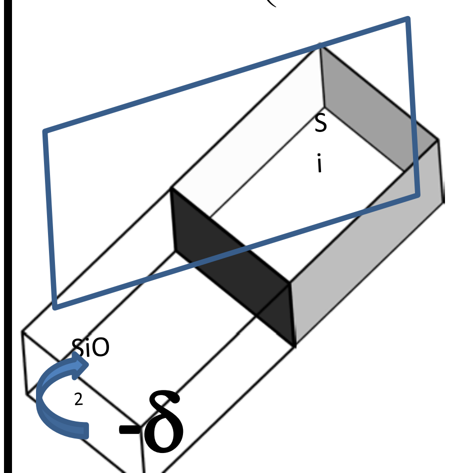

$\delta = 0°$

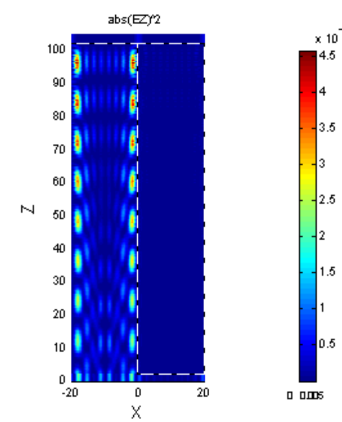
$$J=\begin{pmatrix} 0.3593 + 0.0001i & 0.0000 - 0.0000i \\ 0.0000 - 0.0000i & -0.3654 - 0.0003i \end{pmatrix}$$



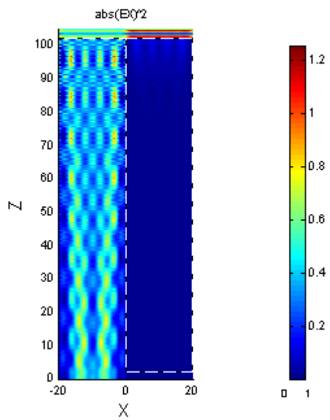

$$J=\begin{pmatrix} 0.3624 + 0.0002i & -0.0031 - 0.0001i \\ 0.0031 + 0.0001i & -0.3624 - 0.0002i \end{pmatrix}$$

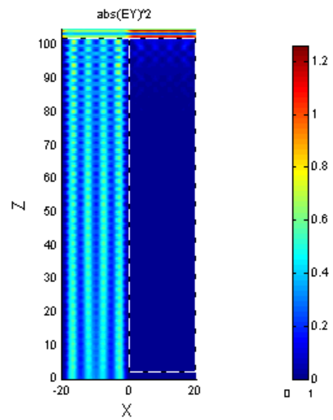


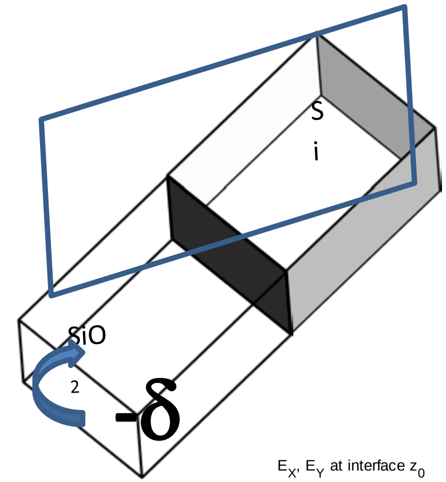



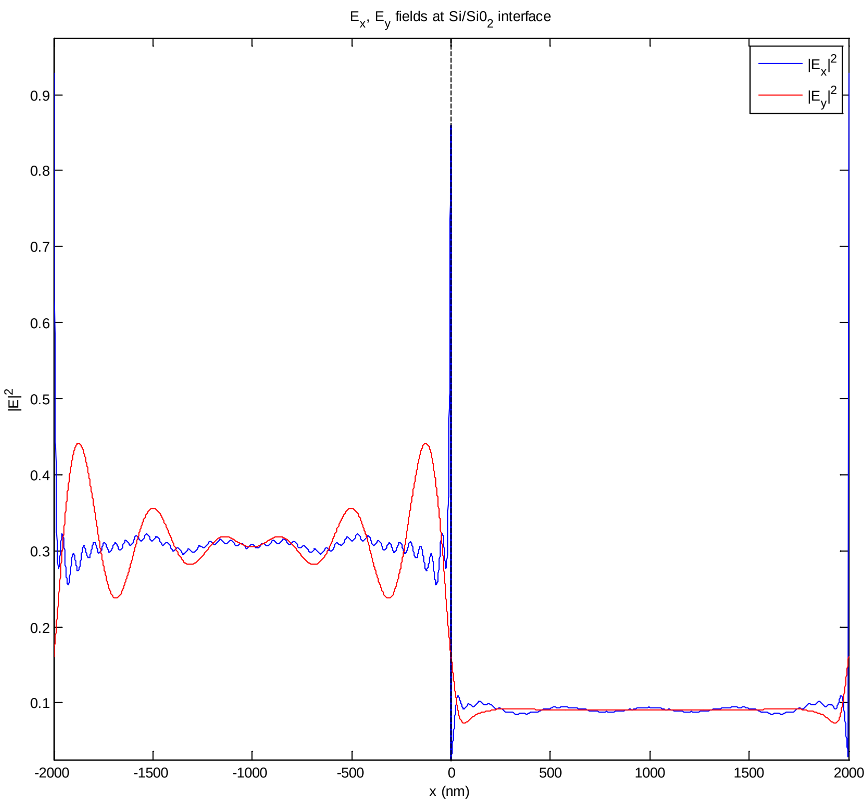
# Numerical Example III

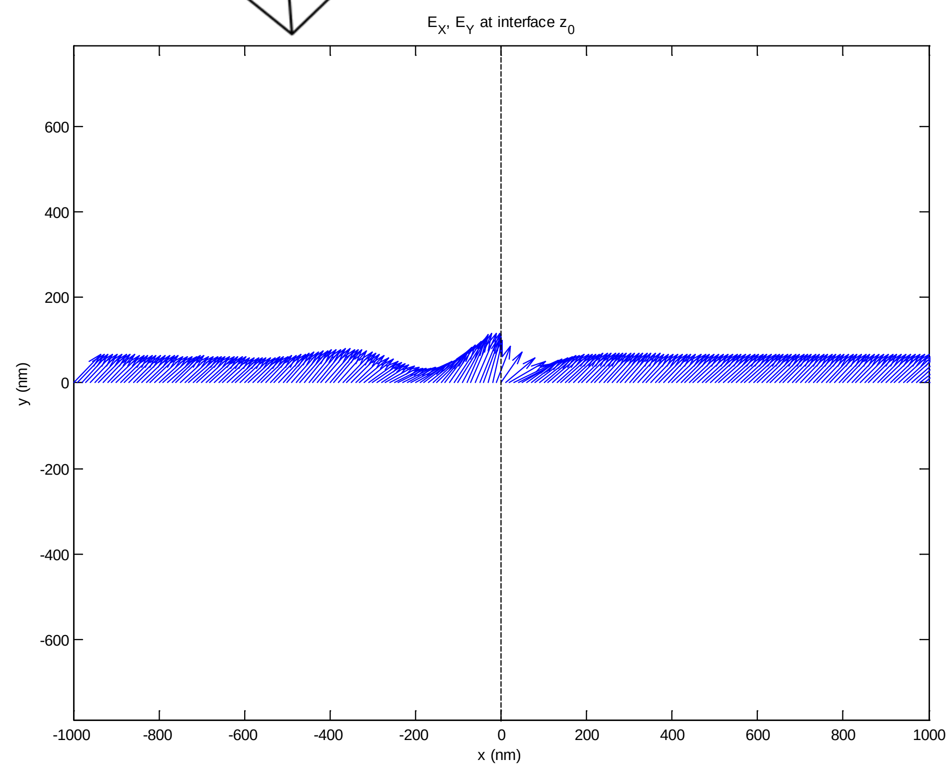
RCWA calculation, normal incidence (θ_in=0 pol-direction (δ) = +45° to IB



# But still totally polarized states, and:

$$\delta = 45°: \quad Jn = \begin{pmatrix} 0.3624 + 0.0002i & 0.0031 + 0.0001i \\ -0.0031 - 0.0001i & -0.3624 - 0.0002i \end{pmatrix}$$

$$\delta = -45°: \quad Jn = \begin{pmatrix} 0.3624 + 0.0002i & -0.0031 - 0.0001i \\ 0.0031 + 0.0001i & -0.3624 - 0.0002i \end{pmatrix}$$

so, (δ) = +45 cancels (δ) = -45°

$$E_p \text{(arising from cross polarization)} =$$

$$r_{ps}(\delta = \chi)E_s + r_{ps}(\delta = -\chi)E_s = 0$$

**For all AOIs as well as for all δs true, so why does in a perfect statistical sample**

**Depolarization**

**occur?**

# But still totally polarized states, and:

$$r_{ps}(\delta = \chi)E_s + r_{ps}(\delta = -\chi)E_s = 0$$

**The measurement system (i.e. mainly the light source) determines, if the fields are superimposed**

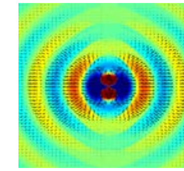
***Coherently***

**Or**

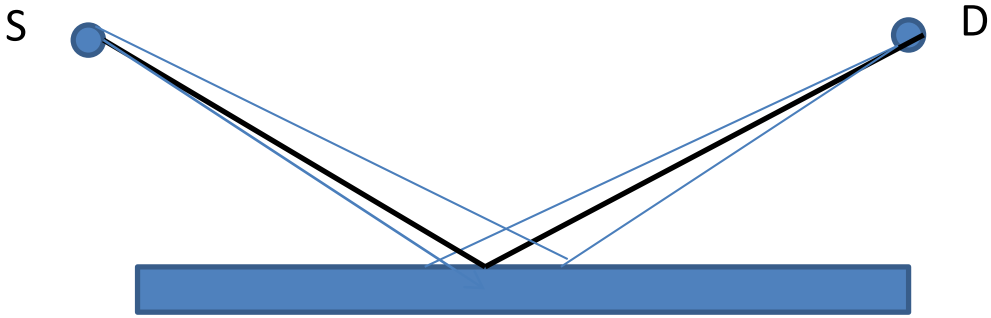
***Incoherently***

***Yielding NO or SOME Depolarisation***

# Decoherence- angle distributions

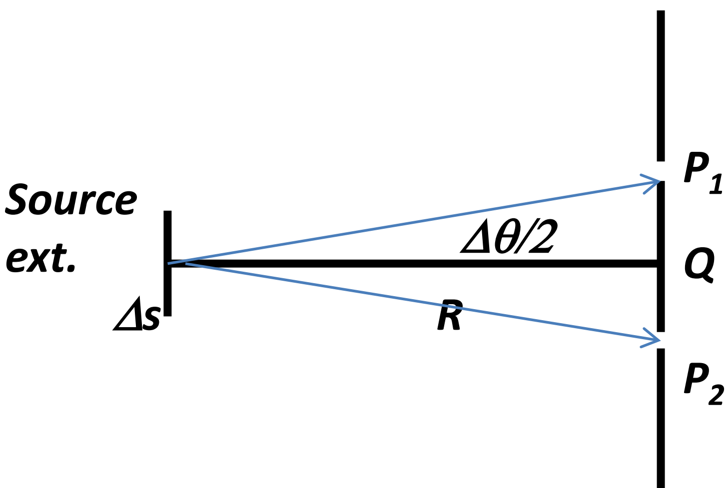


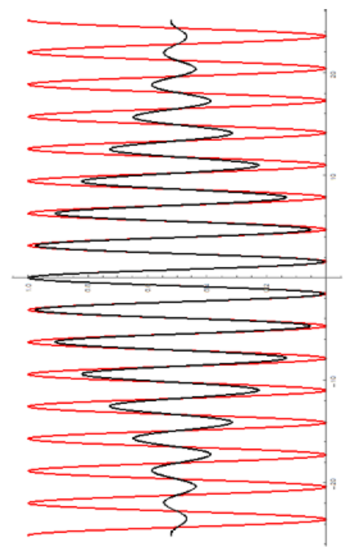
$$\vec{E}_0^2 + \vec{E}_0^2 \cos((\vec{k}_1 - \vec{k}_2)\vec{r} + (\varphi_2 - \varphi_1))$$




- Lets assume quasicoherent radiation: $\Delta\lambda \ll \bar{\lambda}$
- Then, the size of the source counts for interference at the screen:

Transverse coherence area ΔA



$$Q^2 = \Delta A = \frac{R^2 \bar{\lambda}^2}{(\Delta s)^2} \sim (50\mu m)^2$$

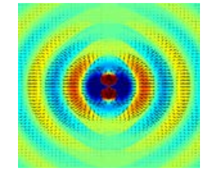
Can be proven: van Cittert – Zernike Theorem

Experiments:

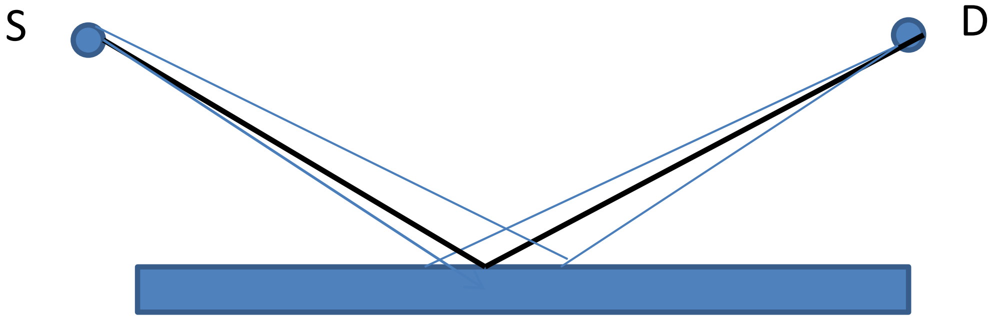
Sun: 0.1 mm^2

Orion star: 6 m^2

# Decoherence



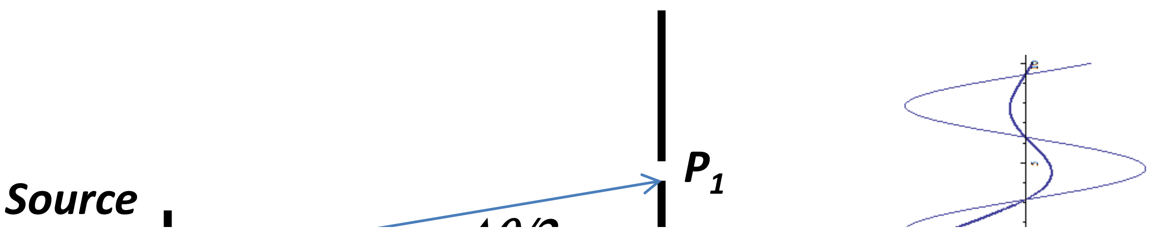
$$\overrightarrow{E}_0^2 + \overrightarrow{E}_0^2 \cos(\vec{k}_1\vec{r} - \vec{k}_2\vec{r} + (\varphi_2 - \varphi_1))$$




- Lets assume quasicoherent radiation: $\Delta\lambda \ll \overline{\lambda}$
- Then, the size of the source counts for interference at the screen:

Transverse coherence area ΔA

$$\Delta A = \frac{R^2 \overline{\lambda}^2}{(\Delta s)^2} \sim (10\mu m)^2$$




**Why is the measured depolarization so small?**

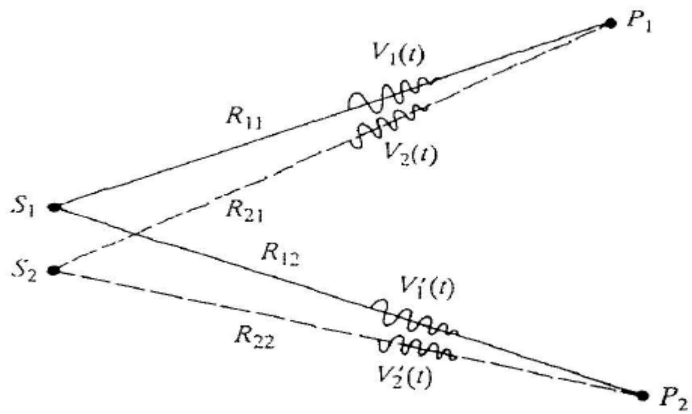
# Decoherence- some mathematics

**„Optical coherence and quantum optics“ L.Mandel and E. Wolf, Cam. UP (1995)**

***„Statistical Optics“, Joseph W. Goodman, Wiley (2000)***

**1. Coherence length does not count „strongly“:**

*“..the fields at $P_1$ and $P_2$, represented by equations (4.2-10), will indeed be strongly correlated. Hence we see that, even though the two sources $S_1$ and $S_2$ are statistically independent, they give rise to correlations ..these correlations are generated in the process of propagation and superposition.”*



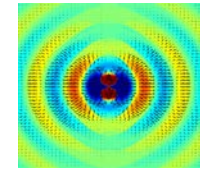

**2. Interference of two stationary light beams as a** **second-order correlation** **phenomenon** $E_{P1}(r,\ t) = K_1\ E(S_1,\ t - R_{11}/c) + K_2\ E(S_2,\ t - R_{21}/c)$

$$I(r,\ t) = E^*(r,\ t)E(r,\ t) = \left|K_1\right|^2\ \left|E(S_1,\ t - R_{11}/c)\right|^2 + \left|K_2\right|^2\ \left|E(S_2,\ t - R_{21}/c)\right|^2 +$$

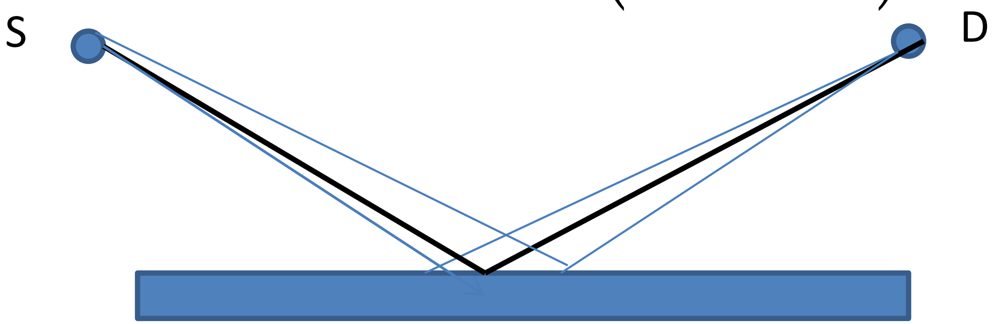
$$2\,\mathrm{Re}(K_1^* K_2 E^*(S_1,\ t - R_{11}/c)E(S_2,\ t - R_{21}/c)) =$$

$$\left|K_1\right|^2\ I(S_1,\ t - R_{11}/c) + \left|K_2\right|^2\ I(S_2,\ t - R_{21}/c) + 2\,\mathrm{Re}\left\{(K_1^* K_2)\Gamma(S_1, S_2, t - t_1, t - t_2)\right\}$$

# Decoherence



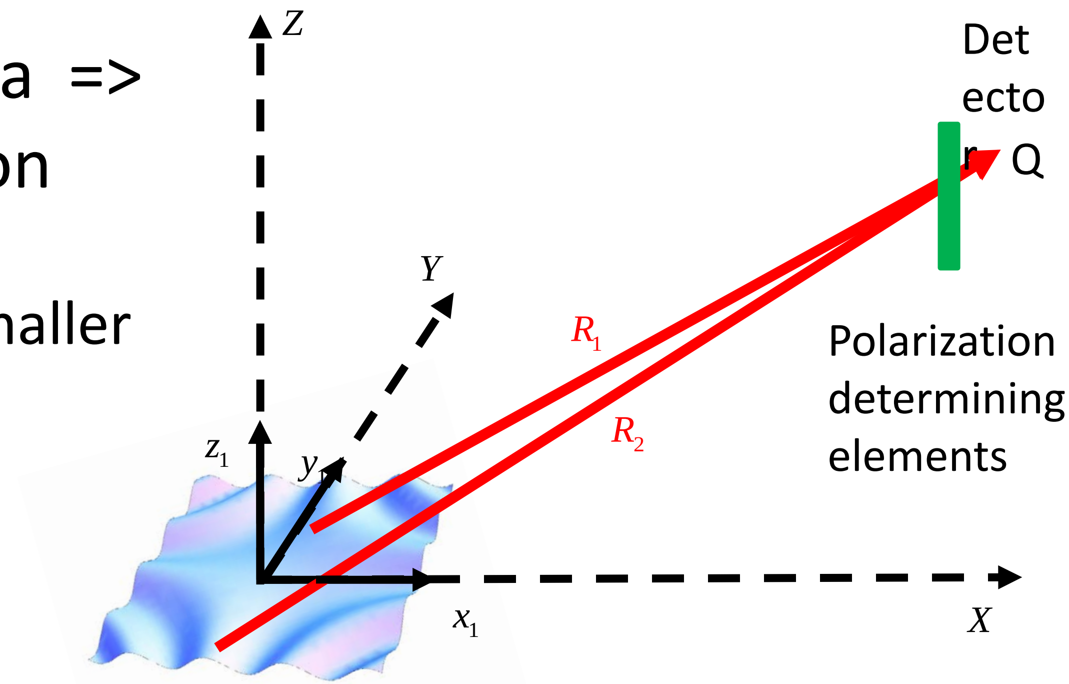
$$\vec{E}_0^{\,2} + \vec{E}_0^{\,2} \cos\left(\left(\vec{k}_1 - \vec{k}_2\right)\vec{r} + (\varphi_2 - \varphi_1)\right)$$




**Spatial Correlation:**

1. Within coherence area => coherent superposition

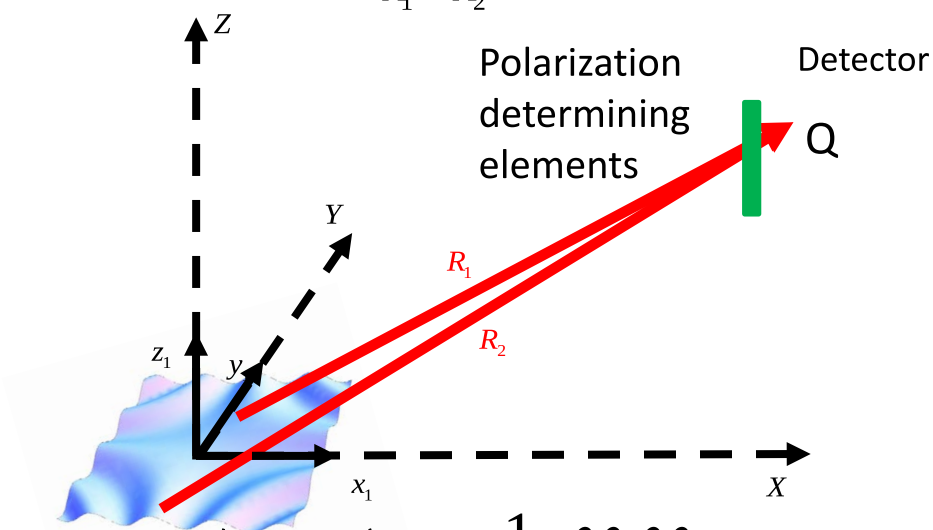
2. Periodic nanostructure smaller than $\sqrt{A}$ =>coherent superposition
3. Two materials ½ ½ => depolarisation



where

# Spatial decoherence- maths

$$I_{(q_i,q_j)}(X,Z) = \iint_{A_1'} \iint_{A_2'} w((x_1', y_1'),(x_2', y_2')) e^{i(\vec{k}(\vec{R}_1' - \vec{R}_2'))} r_{q_i,q_j}(x_1', y_1') r_{q_i,q_j}(x_2', y_2') dS_1' dS_2'$$




__Instrument parameters__

*Spatial coherence:*

Light source area: $\Delta S$. Coherence area: $\Delta A = L^2 = r^2 l^2 / \Delta S$.

Spatial coherence length: $L = \Delta A^{1/2}$. Spot size: $A$.

$$\langle J_k J_m^* \rangle_s = \frac{1}{W} \iint_A \iint_A J_k(x,y) J_m^*(x',y') w\left(\frac{x-x'}{L}, \frac{y-y'}{L}\right) dx'\,dy'\,dx\,dy$$

$$W = \iint_A \iint_A w\left(\frac{x-x'}{L}, \frac{y-y'}{L}\right) dx'\,dy'\,dx\,dy$$

When $L \rightarrow \infty$ (equivalently, D$A \rightarrow \infty$) => *total coherence*

$$\langle J_k J_m^* \rangle_s^{coh} = \frac{1}{A} \iint_A J_k(x,y)\,dx\,dy \;\; \frac{1}{A} \iint_A J_m^*(x',y')\,dx'\,dy' = \langle J_k \rangle_A \langle J_m^* \rangle_A$$

# Spatial decoherence- maths

Instrument parameters

*Spatial coherence:*

Light source area: $\Delta S$. Coherence area:

$\Delta A = L^2 = r^2 l^2/\Delta S$.

Spatial coherence length: $L = \Delta A^{1/2}$. Spot size: $A$.

$$\left\langle J_k J_m^* \right\rangle_s = \frac{1}{W} \iint_A \iint_A J_k(x,y) J_m^*(x',y') w\left(\frac{x-x'}{L}, \frac{y-y'}{L}\right) dx'\,dy'\,dx\,dy$$

When $L \rightarrow \infty$ (equivalently, D$A \rightarrow \infty$) => *total coherence* *"ellipsometry assumption"*

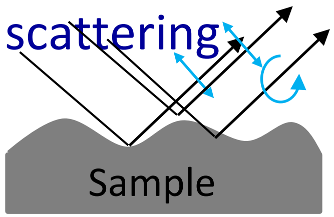
$$\left\langle J_k J_m^* \right\rangle_s^{coh} = \frac{1}{A} \iint_A J_k(x,y)\,dx\,dy \;\; \frac{1}{A} \iint_A J_m^*(x',y')\,dx'dy' = \left\langle J_k \right\rangle_A \left\langle J_m^* \right\rangle_A$$

2. When L $\rightarrow$ 0 (equivalently, DA $\rightarrow$ 0) => total decoherence , MM case, $w \sim \delta(x - x')$

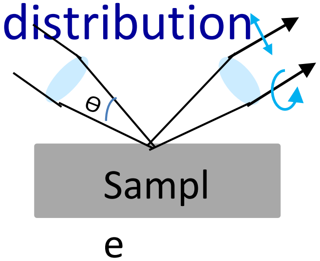
$$\left\langle J_k J_m^* \right\rangle_s^{decoh} = \frac{1}{A} \iint_A J_k(x,y) J_m^*(x,y)\,dx\,dy = \left\langle J_k J_m^* \right\rangle_A$$

# Summary I - the importance of the measurement

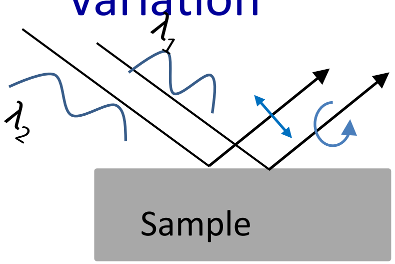
- Surface scattering



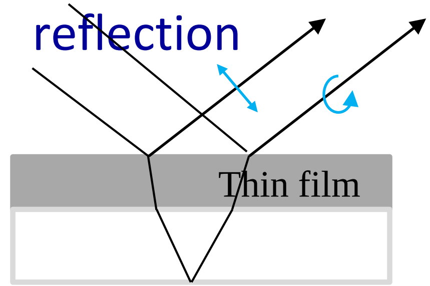

- Incidence angle distribution




- Wavelength variation




- Backside reflection




If samples measured with

- a laser
- and an expanded parallel beam

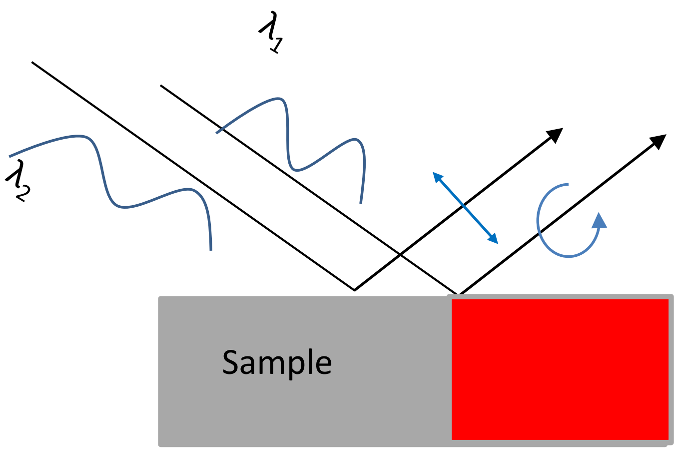
and falling on a

- point-like detector

$\Rightarrow$ NO Decoherence

$\Rightarrow$ NO Depolarisation

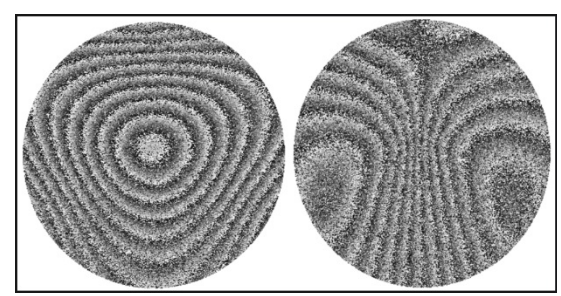
- **Inner boundaries** and thickness inhomogeneity




**Together with finite spatial coherence areas**

**Requirments in addition: Speckles**

Source: hololab.ulg.ac.be



# Summary II Take home messages

- **Inner Boundaries induce Cross Polarization (p<->s) d ~ 10µm!**
- **Stronger contributions from the less polarizable material**
- **The more absorbing the material, the less depolarization**
- **Interference enhances Cross Polarization strongly**
- **Maxwell equations DO NOT describe the loss of INFORMATION**
- ***But the spatial (temporal) coherence properties of ALL of our light sources.***
- ***Decoherence / Depolarization is a statistical process, which depends on the measurement system, too!***
- ***We have to model the detector / source, in order to discriminate sample contributions from experimental setup effects.***

Only three optics books treat the topic of decoherence and depolarization: